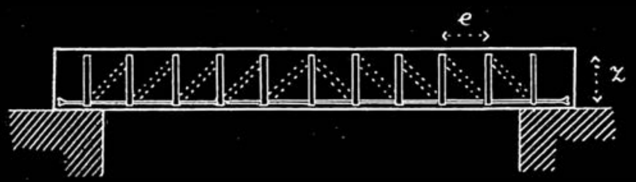
Dutch National Group of *fib* Stufib members meeting
Delft, 2 December 2025

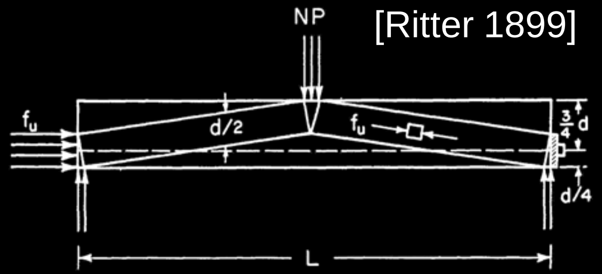
Strut-and-tie models and stress fields: Past, present and future

Walter Kaufmann | Karin Yu

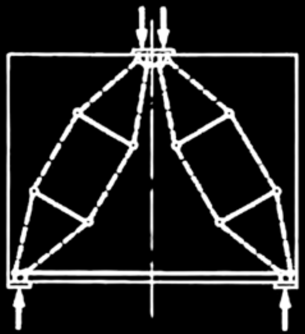
ETH Zürich, Institute of Structural Engineering



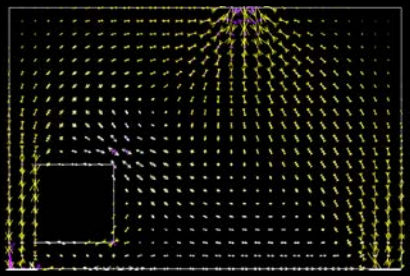
[Ritter 1899]



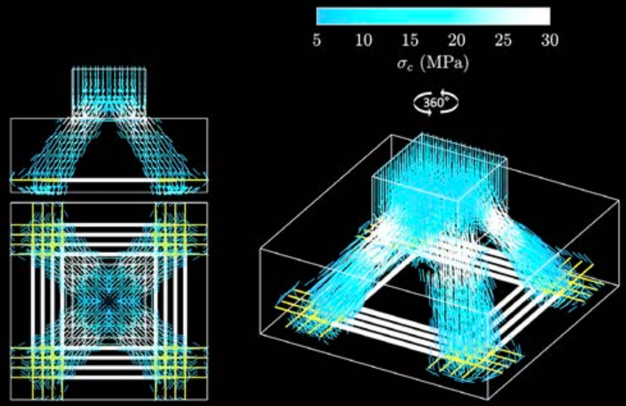
[Drucker 1961]



[Schlaich et al. 1987]

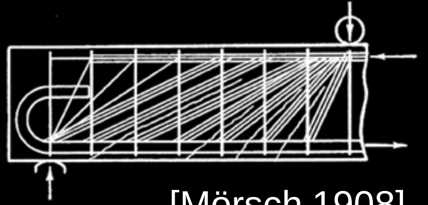


[Kaufmann et al., 2020]

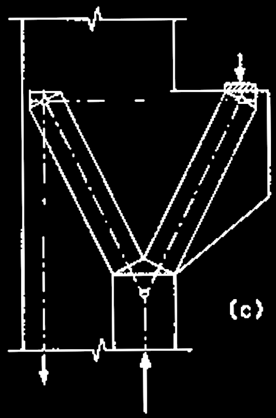


[Meléndez 2017]

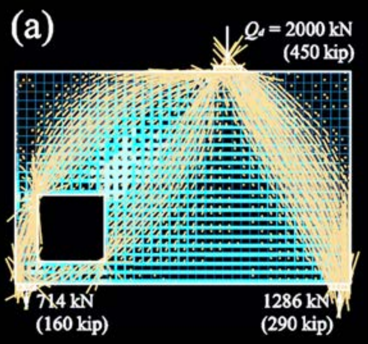
Past **Present** **Future** ?



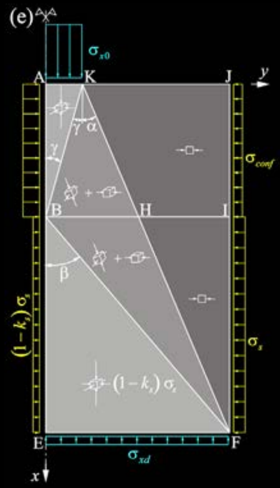
[Mörsch 1908]



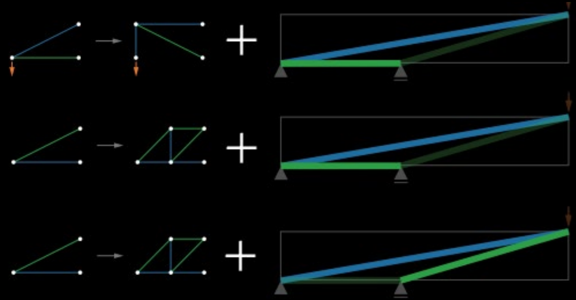
[Marti 1985]



[Muttoni et al. 2015]

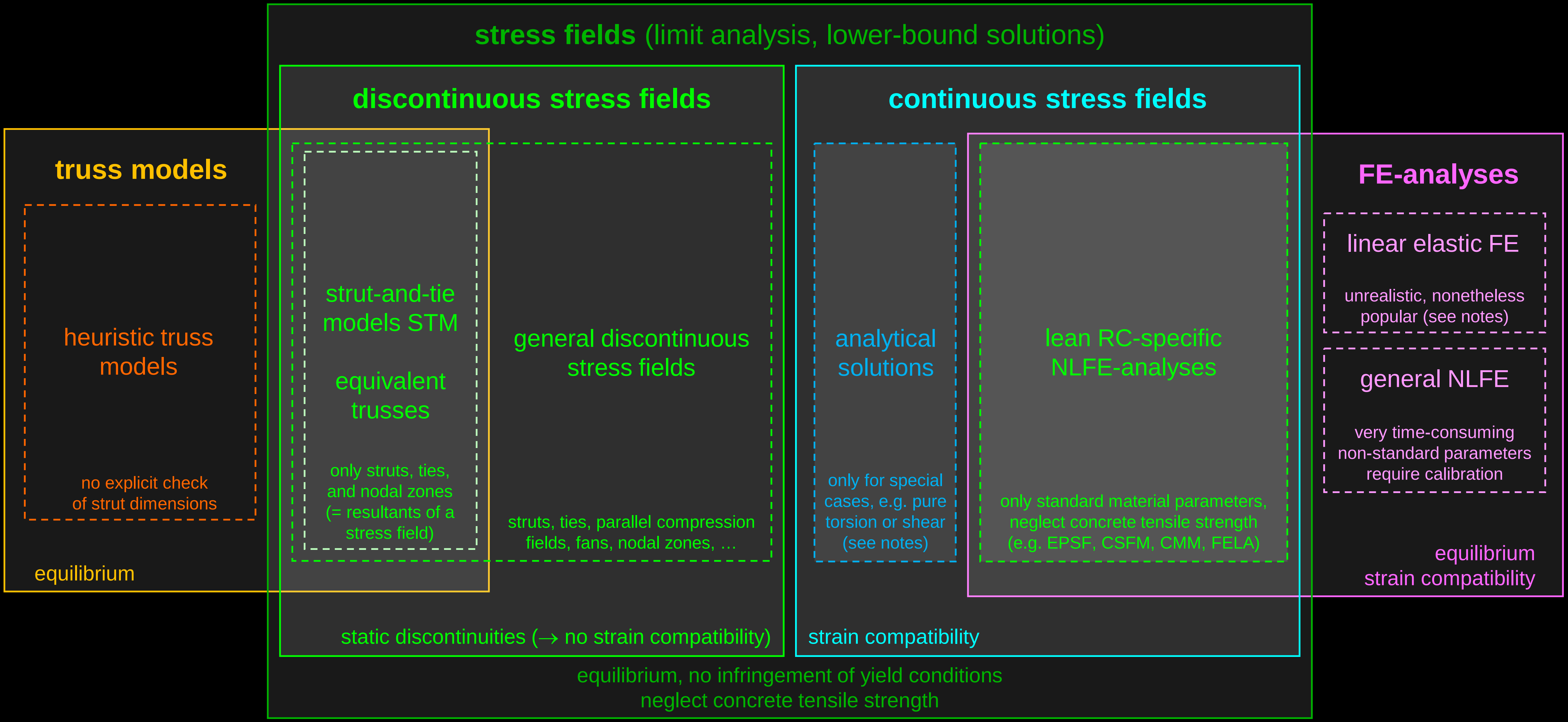


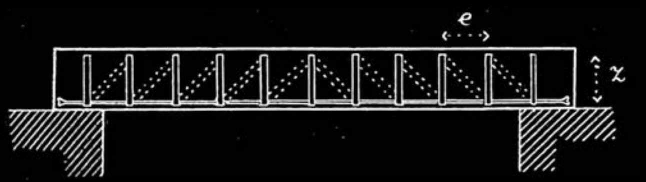
[Markic et al. 2022]



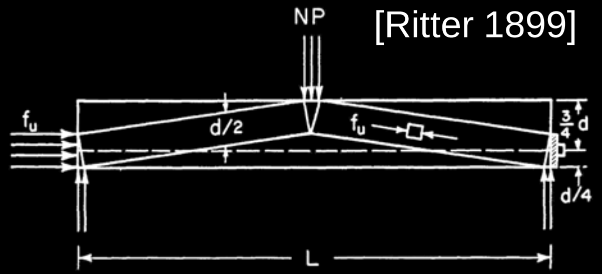
[Yu et al. 2024]

Terminology

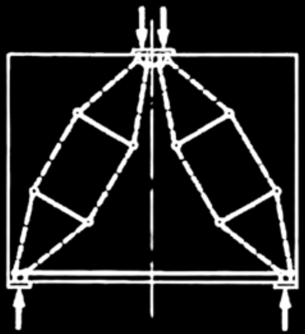




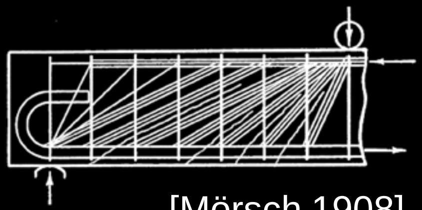
[Ritter 1899]



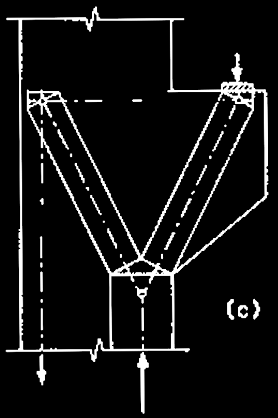
[Drucker 1961]



[Schlaich et al. 1987]



[Mörsch 1908]

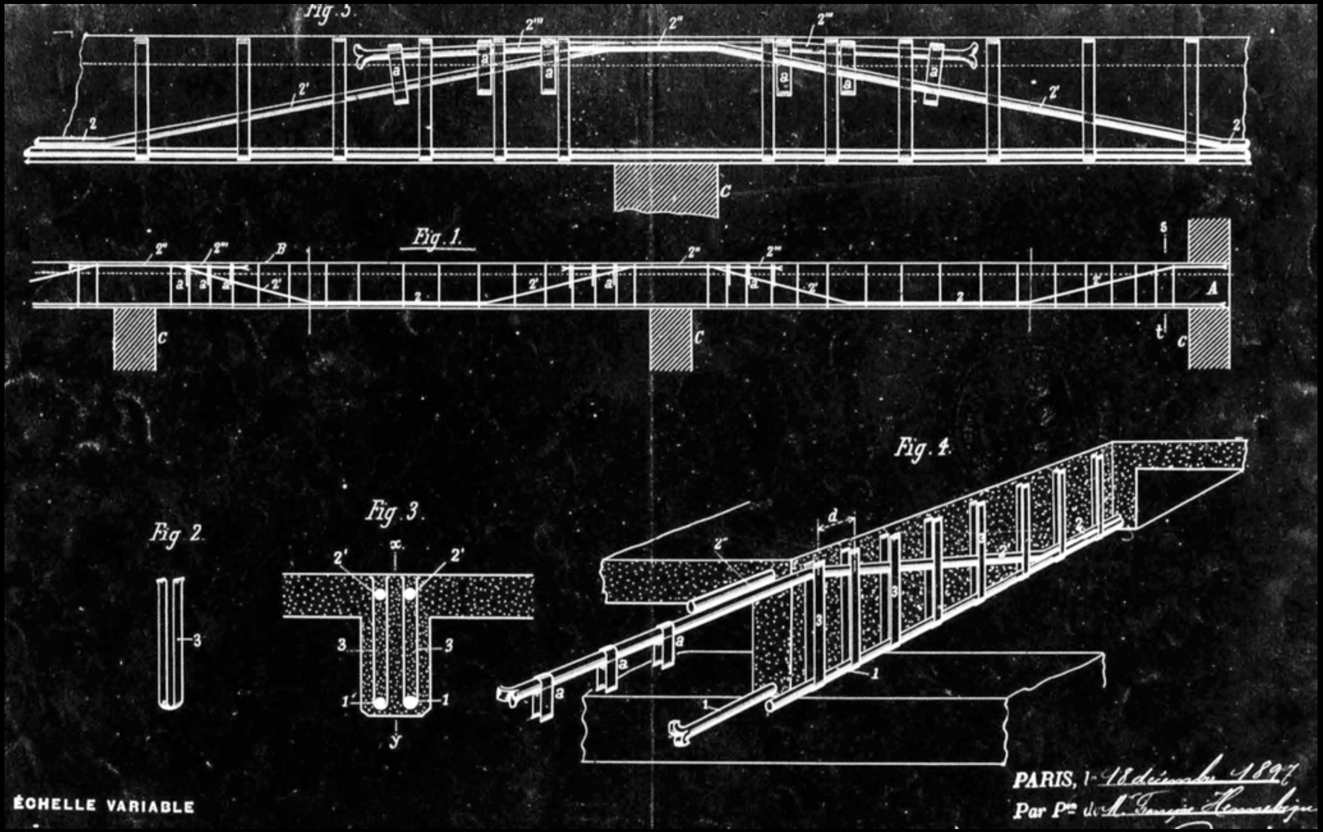


[Marti 1985]

Early days of reinforced concrete: Truss analogy

François Hennebique
1842-1921

Hennebique's patents



[Hennebique 1897]

J'ai perfectionné ces étriers ou entretoises, qui remplacent les tiges de suspension des treillis de poutres métalliques, tandis que le béton qui remplit et forme la partie médiane de la poutre, me donne une succession de rectangles indéformables, auxquels incombe le travail à la compression des bras et montants des treillis des mêmes poutres.

I've perfected these stirrups or diaphragms, which take the place of the suspension rods of metal beam trusses, while the concrete that fills and forms the middle part of the beam gives me a sequence of non-deformable rectangles, to which the compression work of the chords and suspension rods of the trusses of the same beams is assigned.

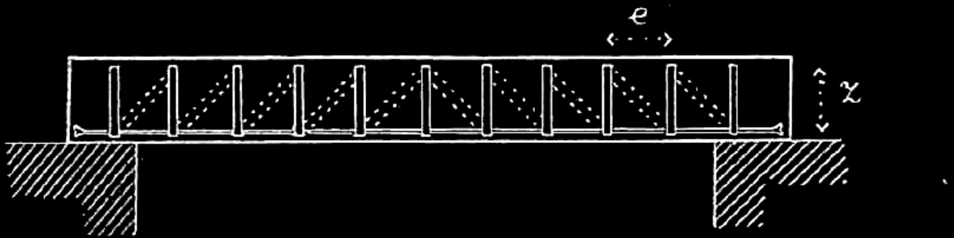
Early days of reinforced concrete: Truss analogy

Karl W. Ritter
1847-1906

Ritter's paper on Hennebique's construction system

Richtung der punktierten Linien wirkend, die Druckstreben darstelle. Diese Linien werden, den Druckkurven entsprechend, unter 45^0 laufend angenommen. Auf Grund dieser An-

Fig. 19.



schauung werden nun die Bügel statisch berechnet und zwar nach der Formel $Q = 2\,\sigma\,b\,d$, worin Q die Scherkraft und b und d Breite und Dicke des Flacheisens bezeichnen. Der Faktor 2 rührt daher, dass jeder Bügel zwei Aeste besitzt.

[Ritter 1899]

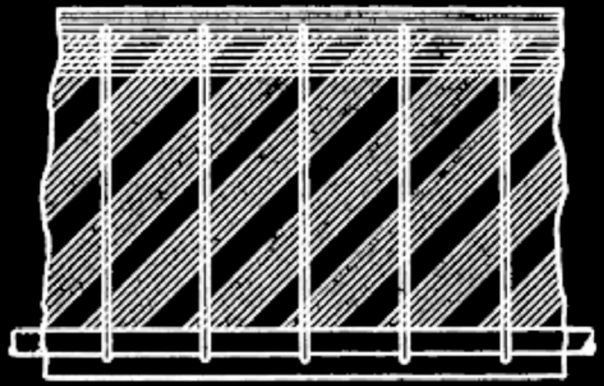
The flat iron stirrups that are placed around the rods are, according to Hennebique's interpretation, supposed to resist the shear forces. Thereby, one usually assumes that the stirrups, together with the (longitudinal) rods and the concrete, form a kind of truss in which the stirrups represent the ties and the concrete, acting in the direction of the dotted lines, the compression struts.

It is difficult to say to what extent the inventor's understanding of the effect of his stirrups is correct. [...] In my opinion, however, the effect of the stirrups is rather that they resist the tensile stresses acting in the direction of the tension curves and prevent premature cracking.

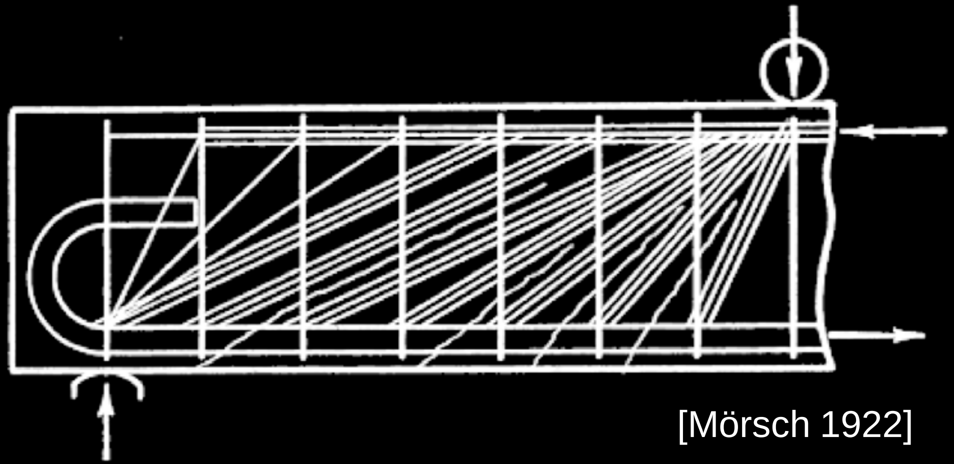
Early days of reinforced concrete: Truss analogy

Emil Mörsch
1872-1950

Mörsch's seminal book(s) on reinforced concrete construction



[Mörsch 1908]

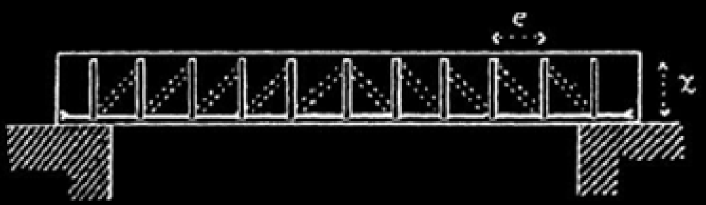


[Mörsch 1922]

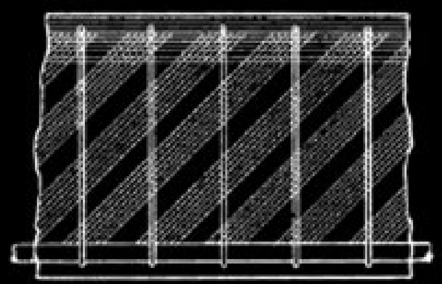
However, if the main reinforcement is designed as described above, the stirrups will only have a subordinate static function and can be regarded as a further safety factor that comes into effect if other structural elements fail. On the other hand, the stirrups are also statically required in the middle part of the beam, where shear forces occur in case of half-sided loading, which are commonly disregarded in structural analyses.

~~Truss analogy = Modern stress fields?~~

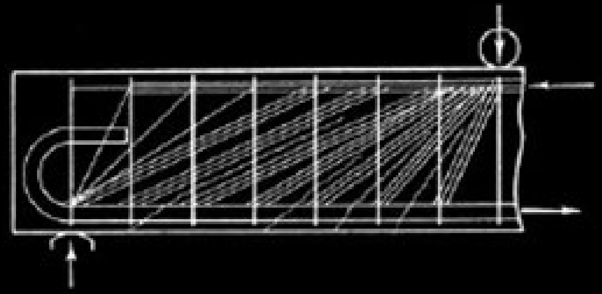
Early truss models (heuristic)



K. W. Ritter, «Die Bauweise Hennebique» (1899)



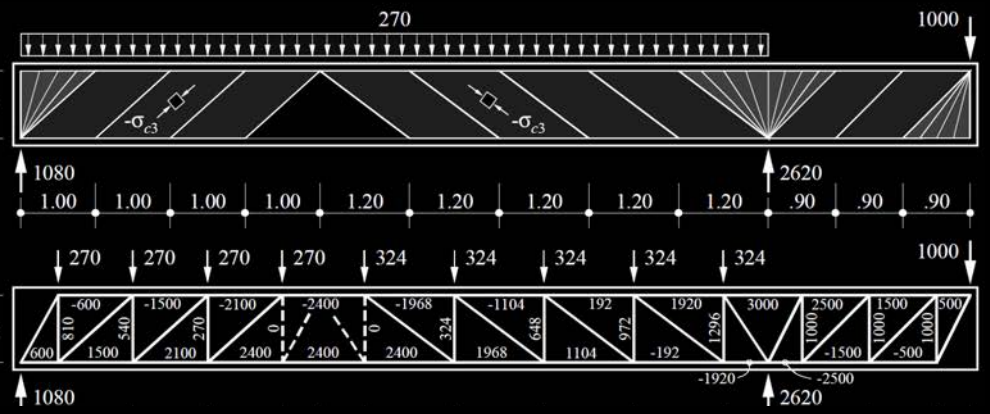
E. Mörsch,

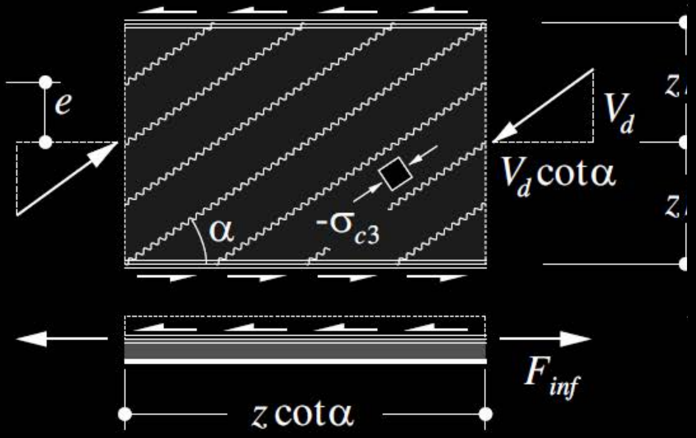


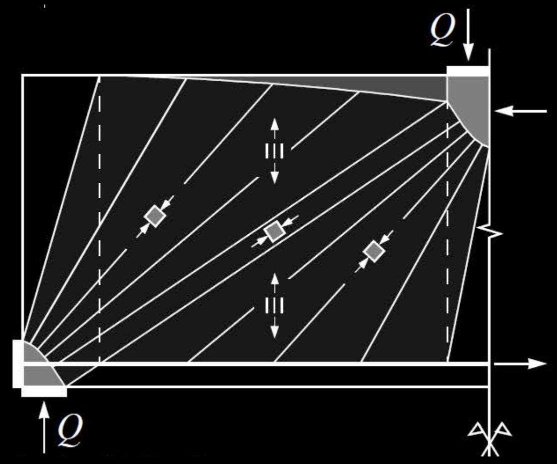
E. Mörsch, «Der Eisenbetonbau» (1922)

solid foundation only
in the 1960s by the
theory of plasticity

Modern stress fields / strut-and-tie models

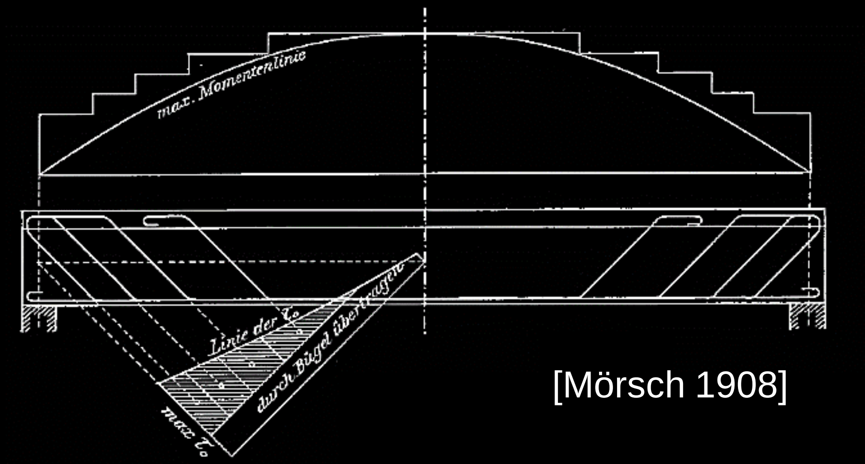




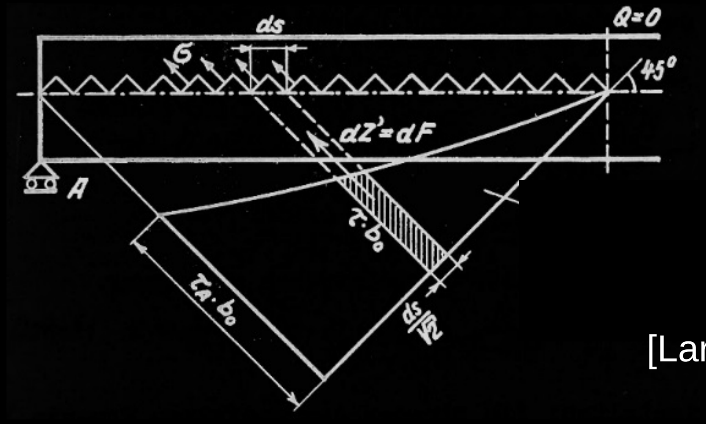


Semi-empirical shear design

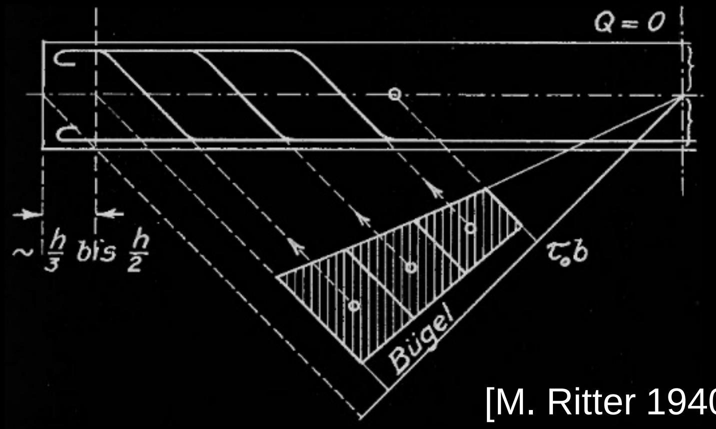
Design based on principal tensile stresses / Kani's works (no stirrups) / …



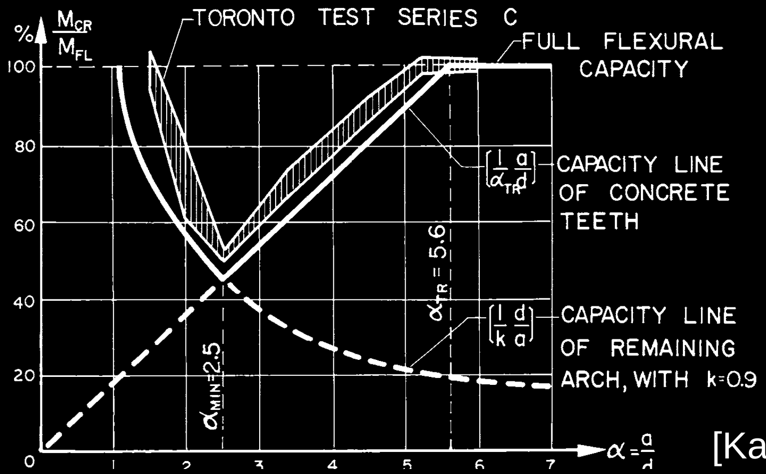
[Mörsch 1908]



[Lardy 1951]



[M. Ritter 1940]



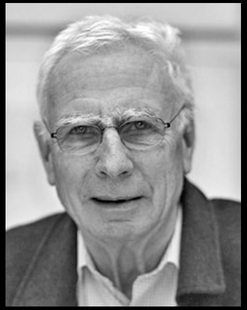
[Kani 1964]

Emil Mörsch
1872-1950

Max Ritter
1884-1946

Pierre Lardy
1903-1958

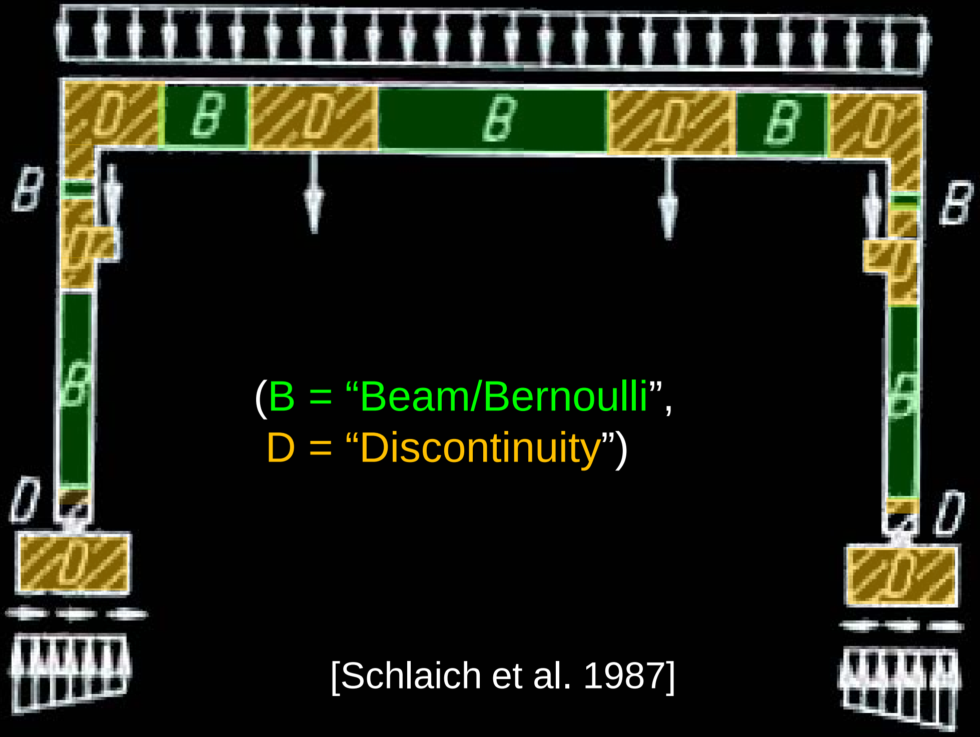
Heuristic strut-and-tie models



Jörg Schlaich
1934-2021

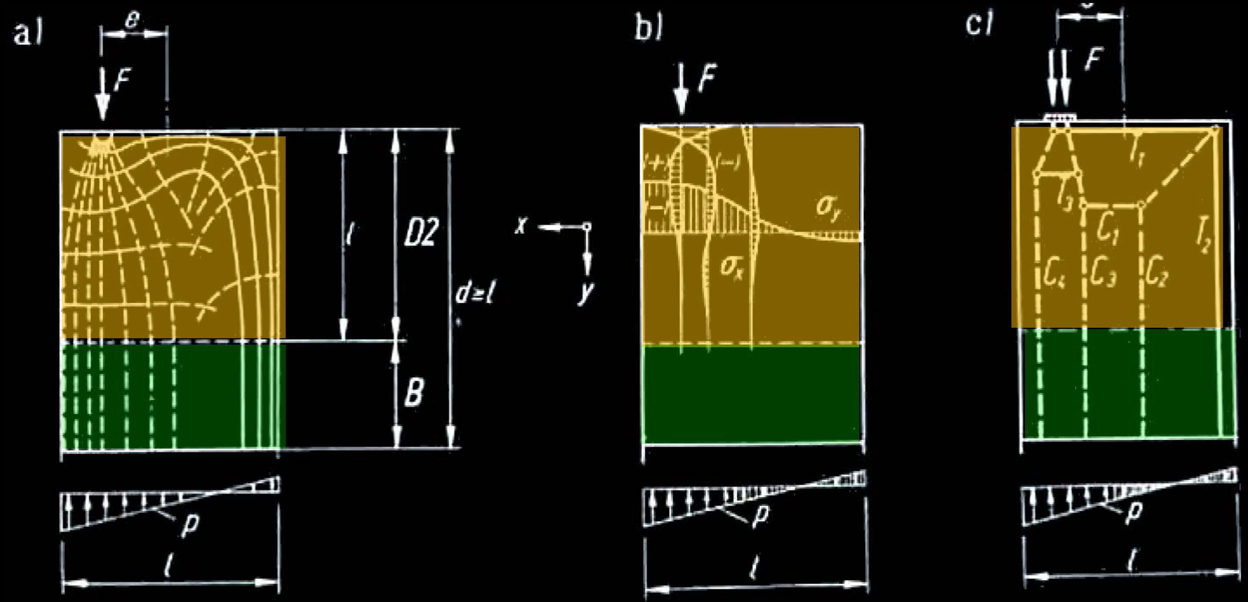
Schlaich et al.'s seminal works: design based on heuristic truss models

Terminology:
B-Regions: geometry and loading continuous → plane sections, sectional design ok
D-Regions: geometry or loading discontinuous → sectional design not ok

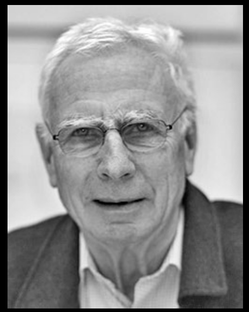


(B = "Beam/Bernoulli",
D = "Discontinuity")

[Schlaich et al. 1987]



Heuristic strut-and-tie models

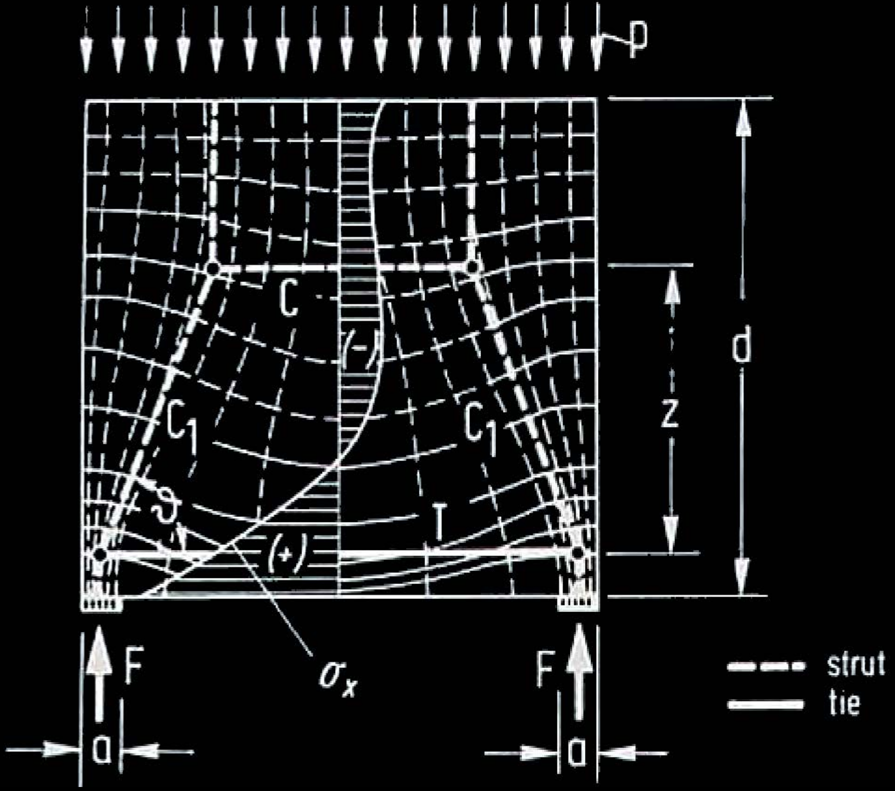


Jörg Schlaich
1934-2021

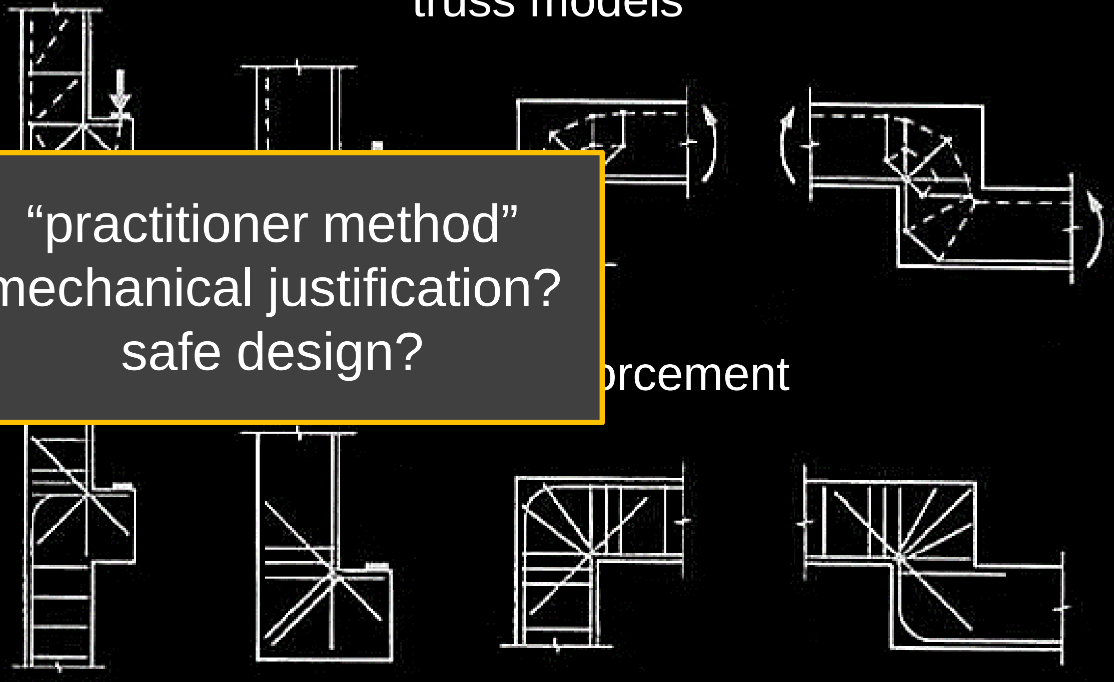
and co-authors
K. Schäfer,
M. Jennewein,
D. Weischede,
etc.

adopted by many
others:
J.G. MacGregor
etc.

Schlaich et al.’s seminal works: design based on heuristic truss models



truss models



“practitioner method”
mechanical justification?
safe design?

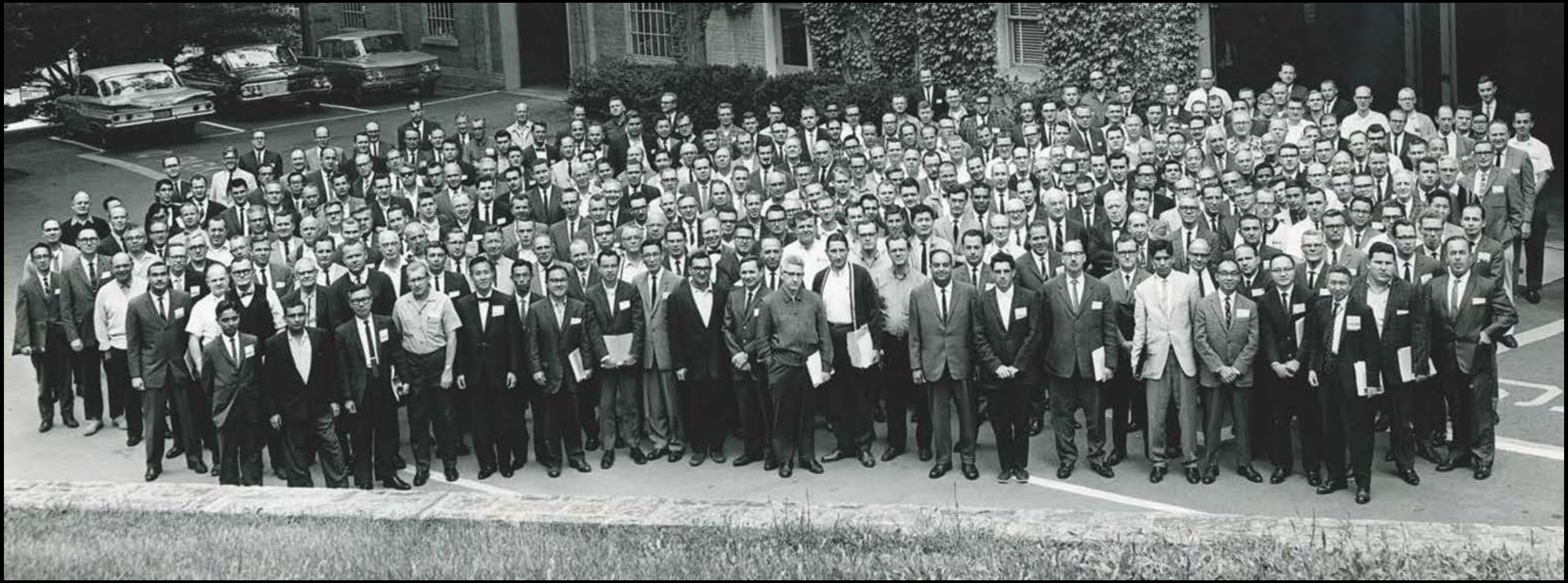
[Schlaich et al. 1987]

truss layout based
on linear elastic solution

primary interest: force flow and reinforcement design;
strut dimensions of secondary interest (rarely checked)

Plasticity theory

Limit analysis: First application in steel structures and soil mechanics



[Plasticity conference at Lehigh University, 1965 (photo from Bruno Thürlimann's archive)]

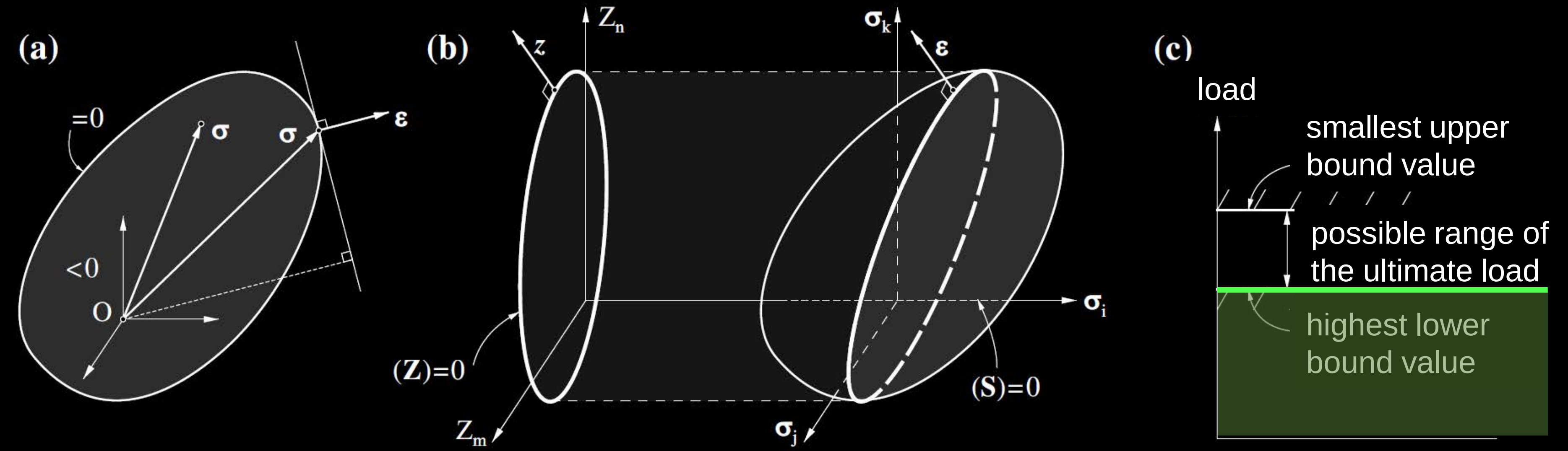
William Prager
1903-1980

Daniel C. Drucker
1918-2001

and many others,
A.E. Green,
R. Hill, W.T. Koiter,
R. Shield, etc.

Plasticity theory

Limit analysis: Lower-bound theorem



General assumptions:
- convexity and orthogonality (↔ maximum dissipation)
- perfectly plastic behaviour (↔ ductility)

Lower-bound theorem:
- satisfy equilibrium and static boundary conditions
- do not infringe yield condition

safe design

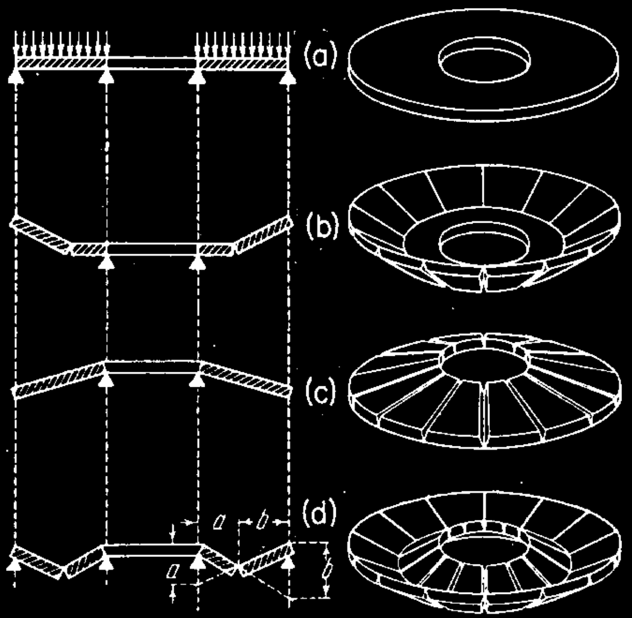
William Prager
1903-1980

Daniel C. Drucker
1918-2001

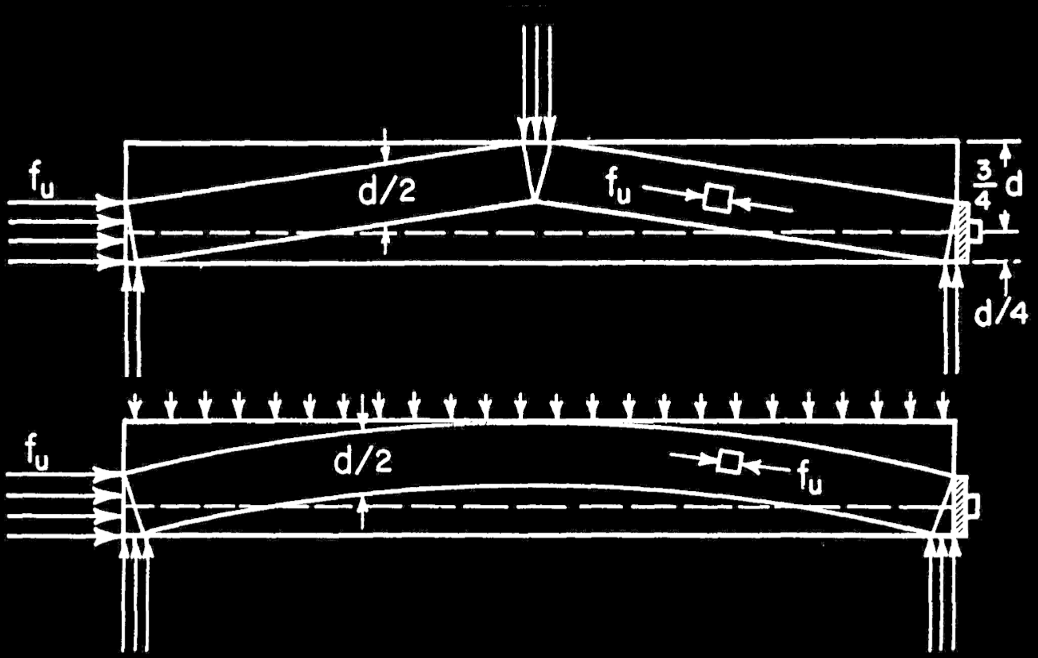
Alexey A. Gvozdev
1897-1986

Plasticity-based models for reinforced concrete

Limit analysis: Application to structural concrete



[Gvozdev 1936]



[Drucker 1961]

William Prager
1903-1980

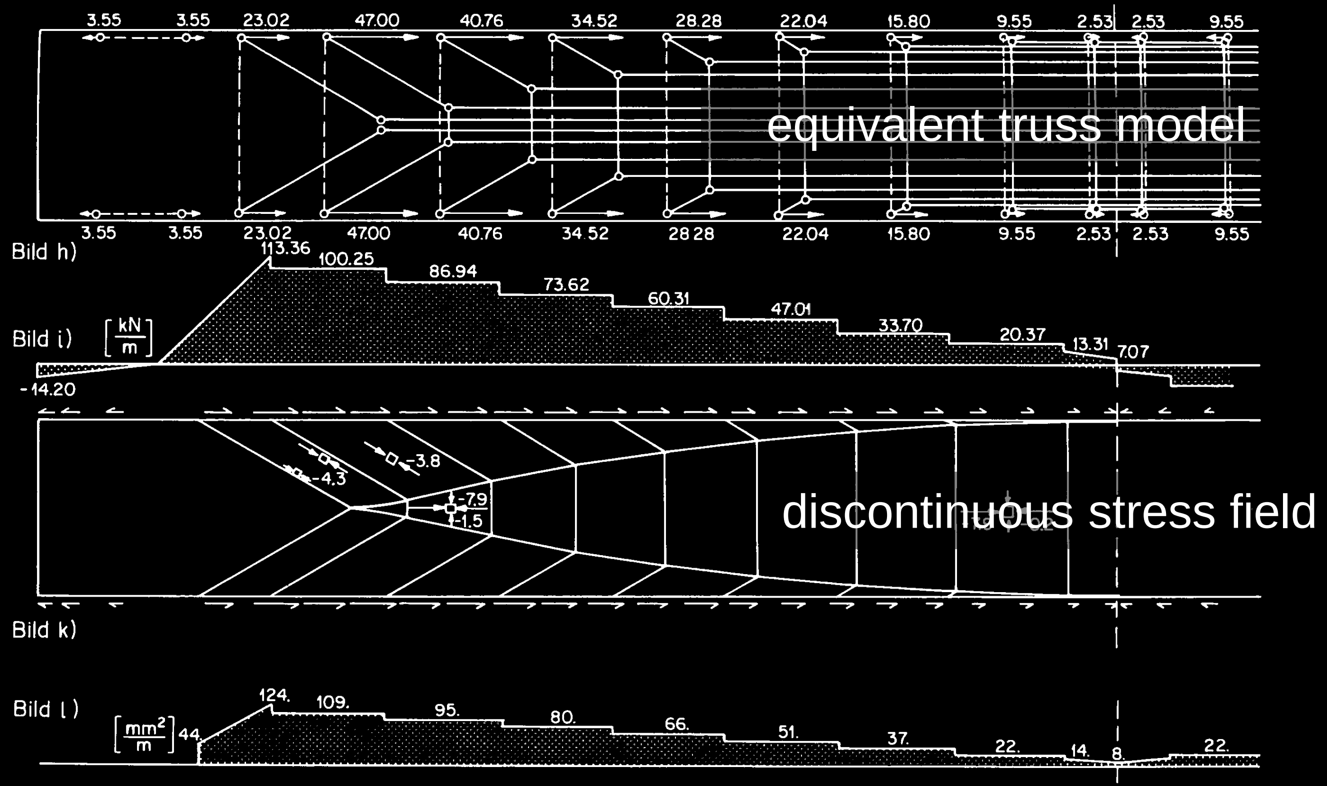
Daniel C. Drucker
1918-2001

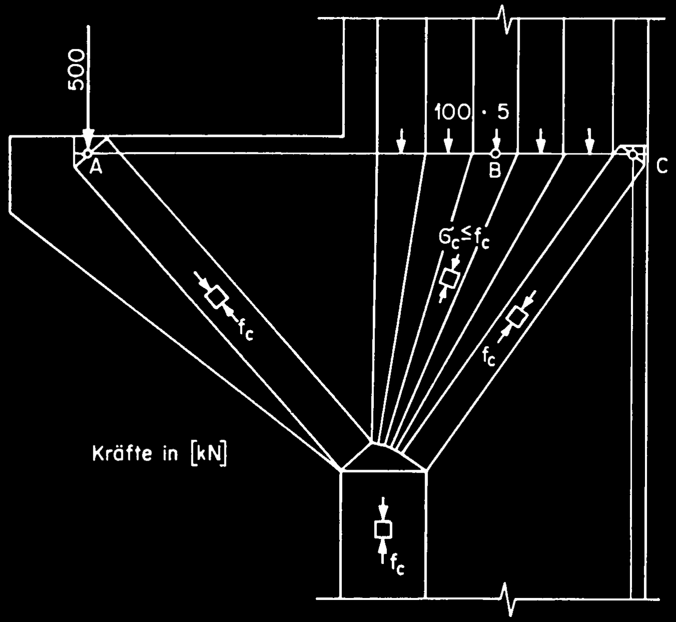
Alexey A. Gvozdev
1897-1986

Many others, e.g. A. Sawczuk, T. Jaeger, R.H. Wood, G.S.T. Armer, …

Plasticity-based models – Zürich school

Limit analysis: Application to structural concrete

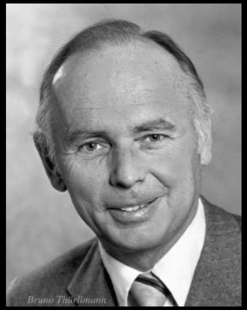




[Thürlimann, Marti et al. (1984)]

truss / stress field layout based on engineering judgement

analogy of stress fields and equivalent truss models
strut dimensions based on *fc*

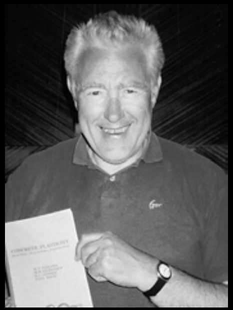


Bruno Thürlimann
1923-2008
Prof. ETH 1960-1990

Peter Marti
*1949
Prof. ETH 1990-2014

and many others:
Paul Lampert,
Peter Müller,
Josef Grob,
…

Plasticity-based models – Danish / British schools

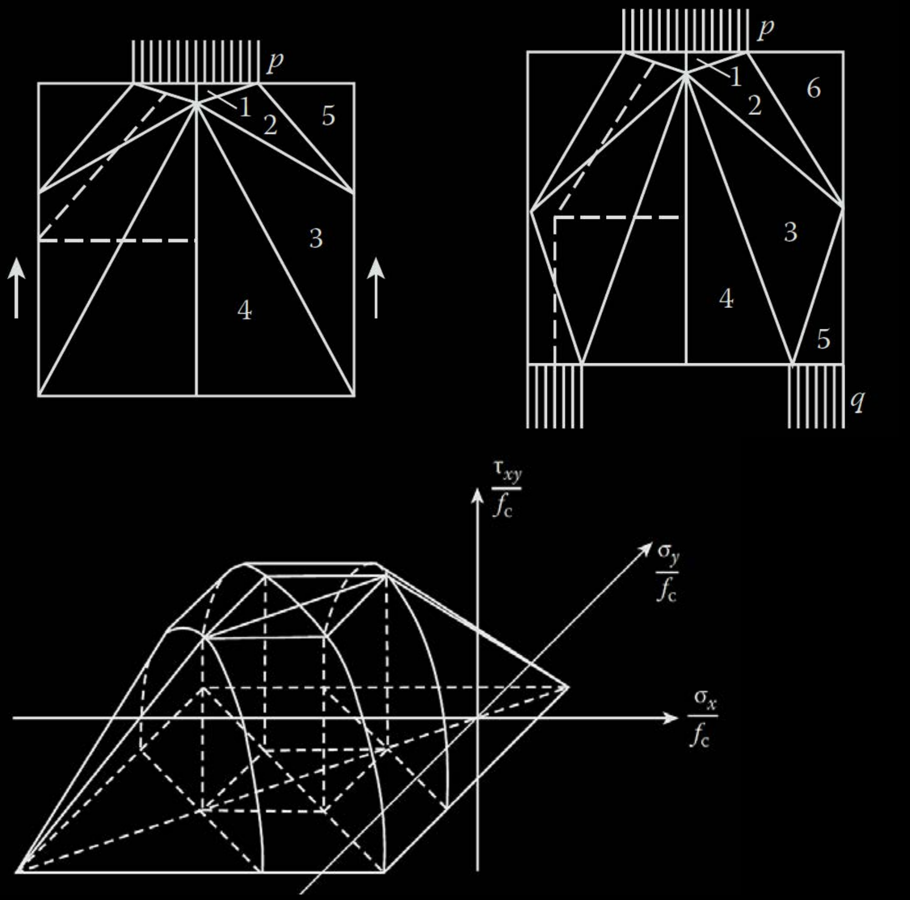


Mogens P. Nielsen
*1935

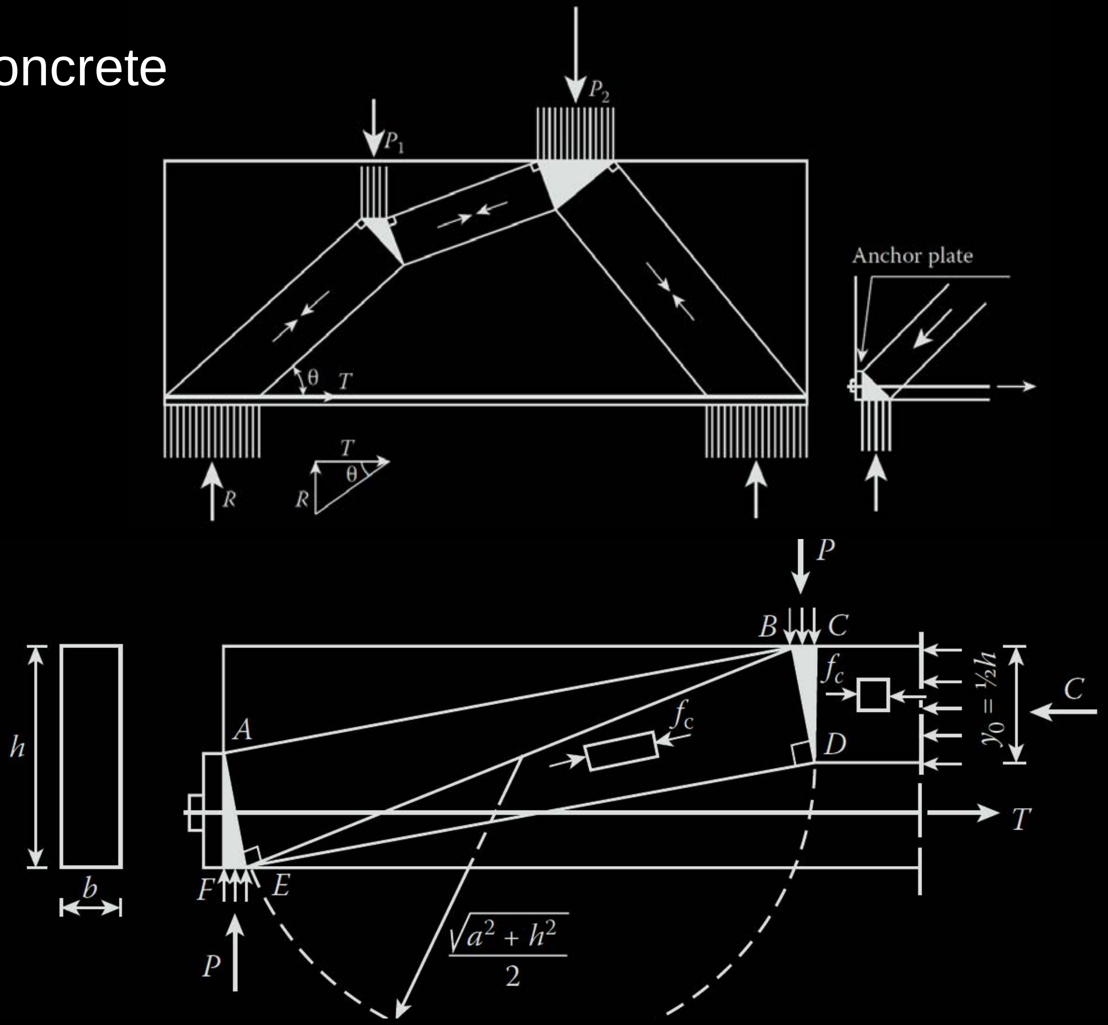
and many others:
Douglas C. Clyde,
Wai-Fah Chen,
Christopher T. Morley,
Michael W. Braestrup,
Linh Cao Hoang,
…

partly inspired by plasticity in soil mechanics

Limit analysis: Application to structural concrete



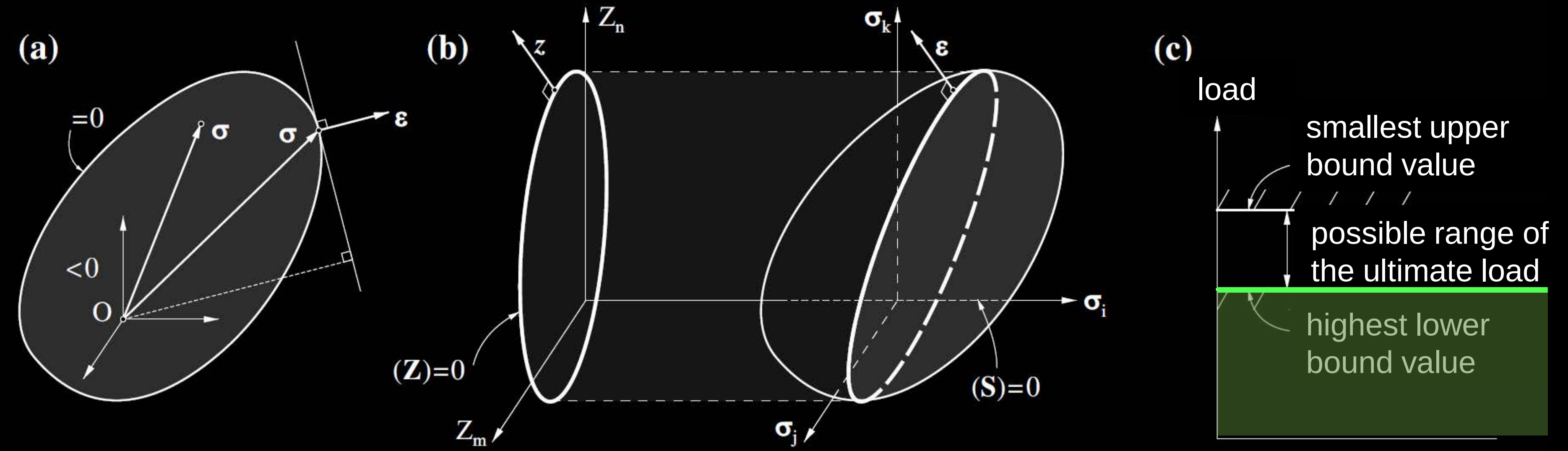
[Nielsen 1969 / Nielsen and Hoang 2011]



[Nielsen 1984 / Nielsen and Hoang 2011]

Limited ductility

Limit analysis: Lower-bound theorem



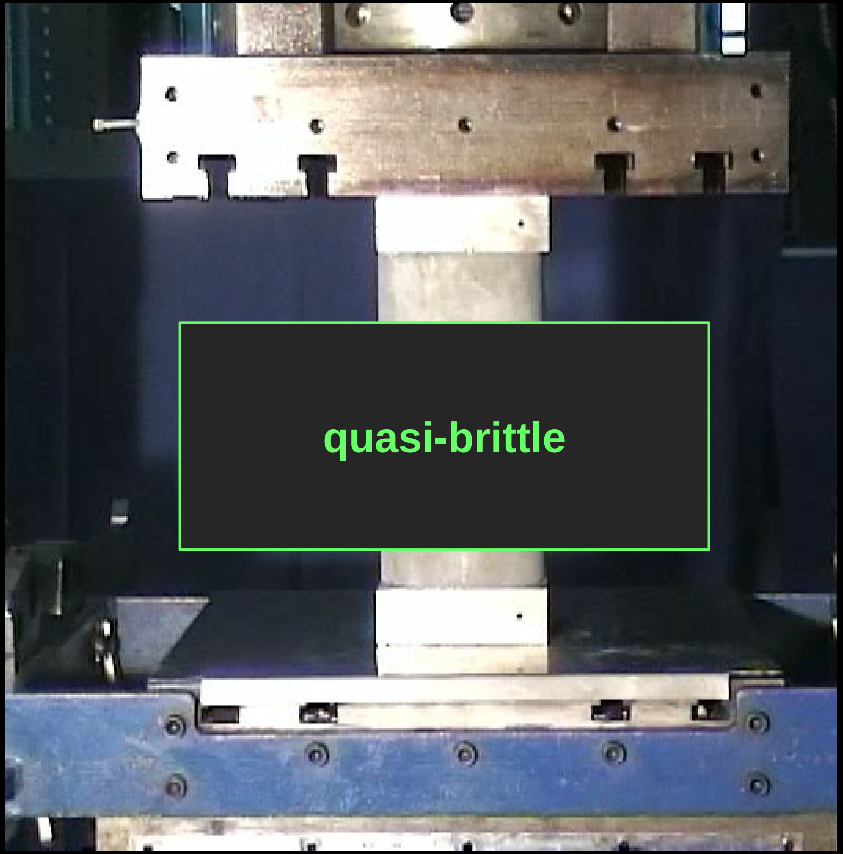
General assumptions:
- convexity and orthogonality (↔ maximum dissipation)
- perfectly plastic behaviour (↔ ductility)

Lower-bound theorem:
- satisfy equilibrium and static boundary conditions
- do not infringe yield condition

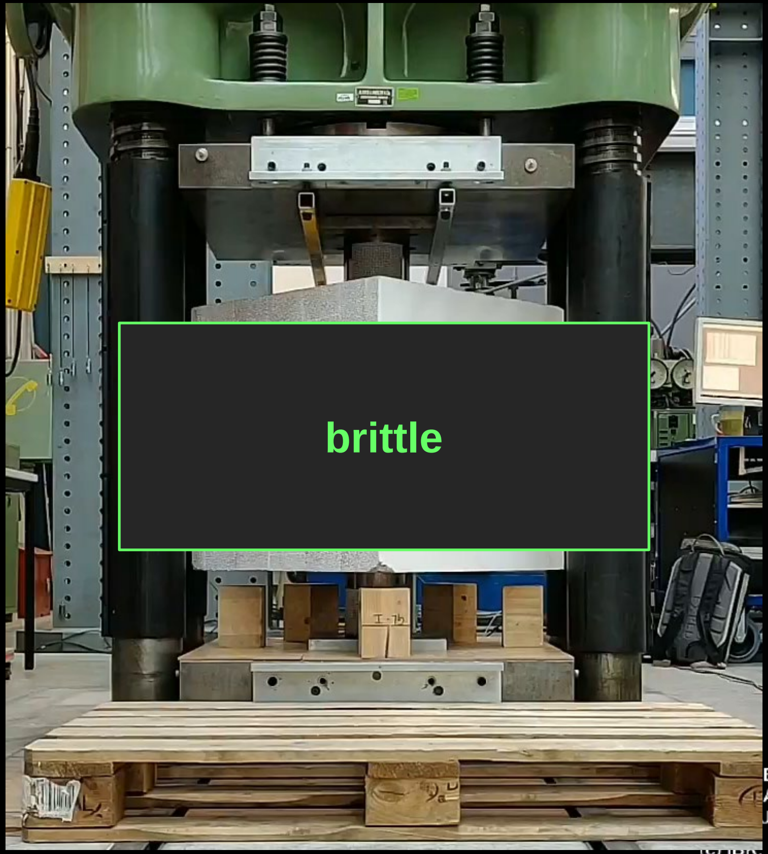
safe design
???

Limited ductility

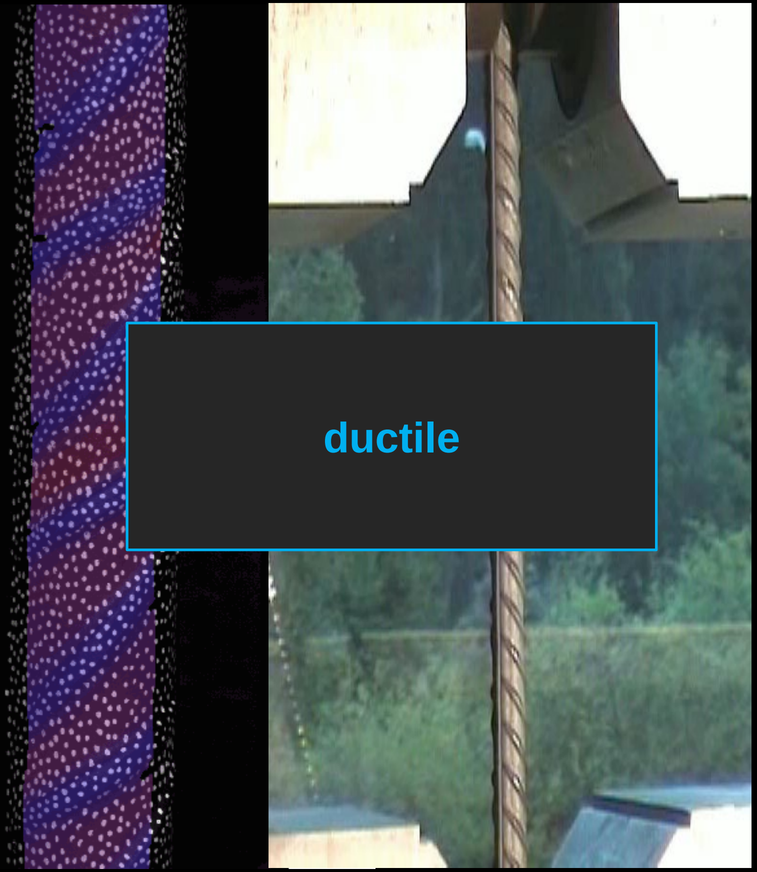
Concrete in compression



Concrete in (indirect) tension



Reinforcing steel



Limited ductility

Prerequisites of the theorems of limit analysis

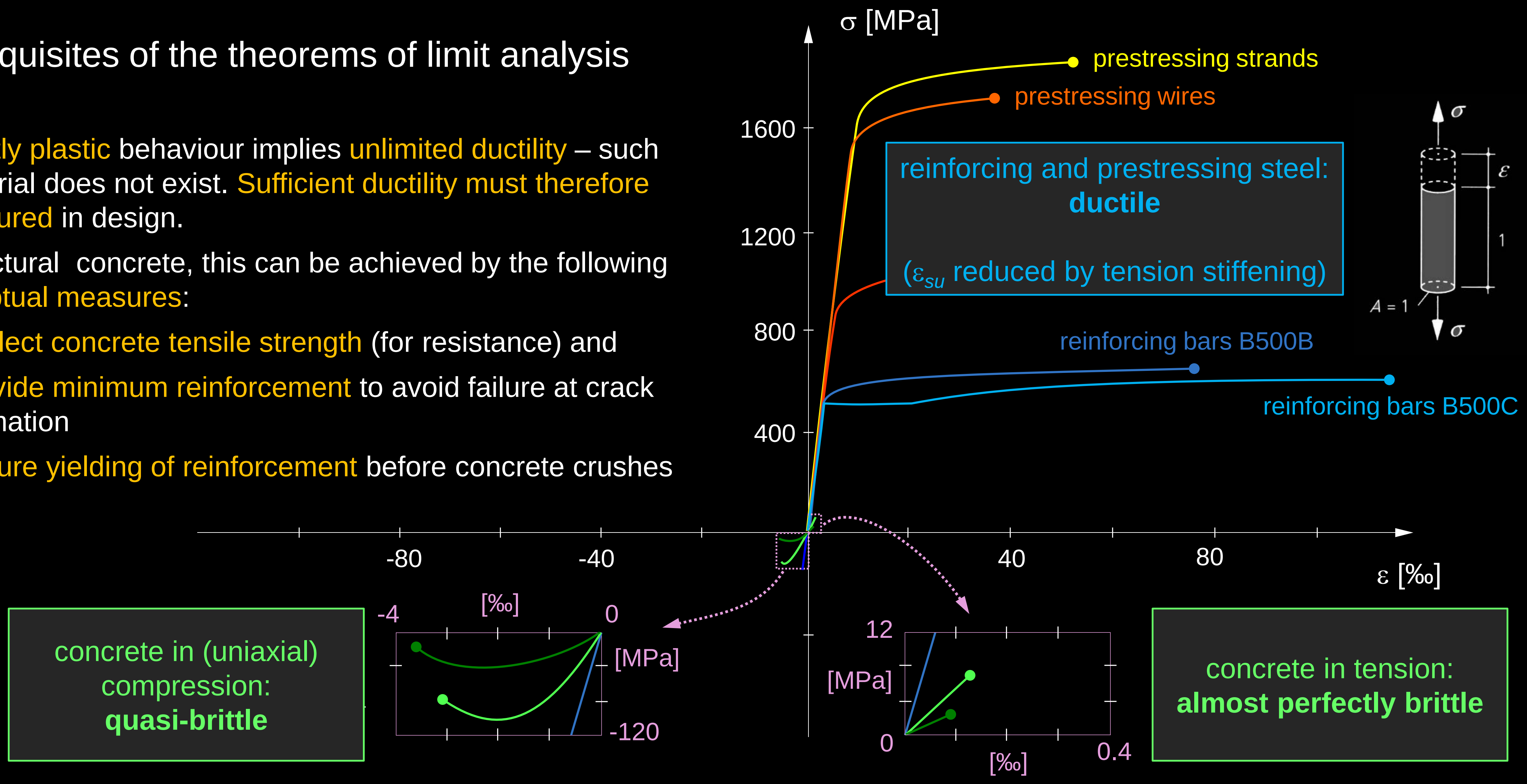
Perfectly plastic behaviour implies unlimited ductility – such a material does not exist. Sufficient ductility must therefore be ensured in design.

In structural concrete, this can be achieved by the following conceptual measures:

→ neglect concrete tensile strength (for resistance) and

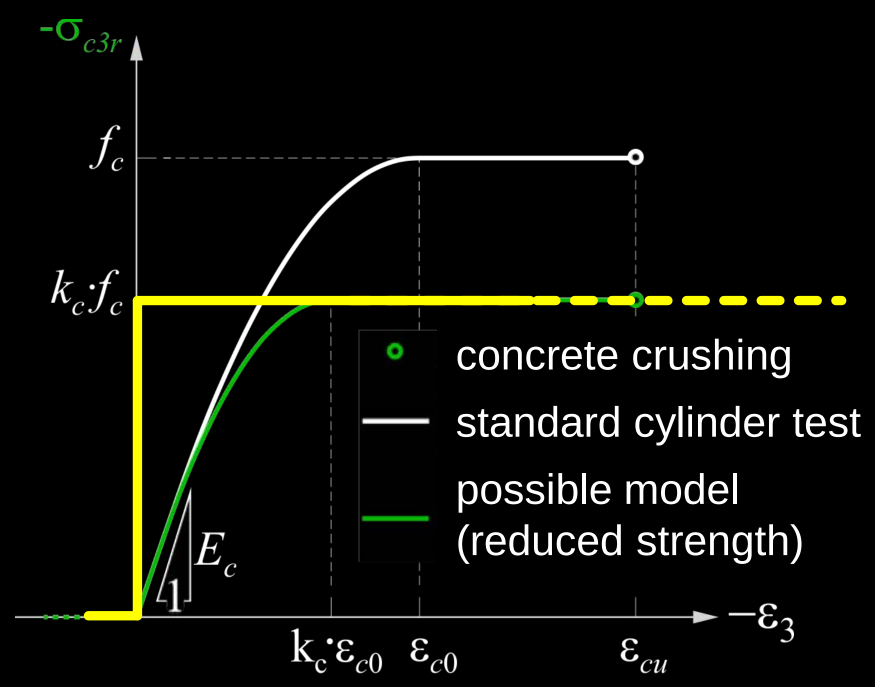
→ provide minimum reinforcement to avoid failure at crack formation

→ ensure yielding of reinforcement before concrete crushes



Limited ductility vs perfectly plastic behaviour

Concrete in compression



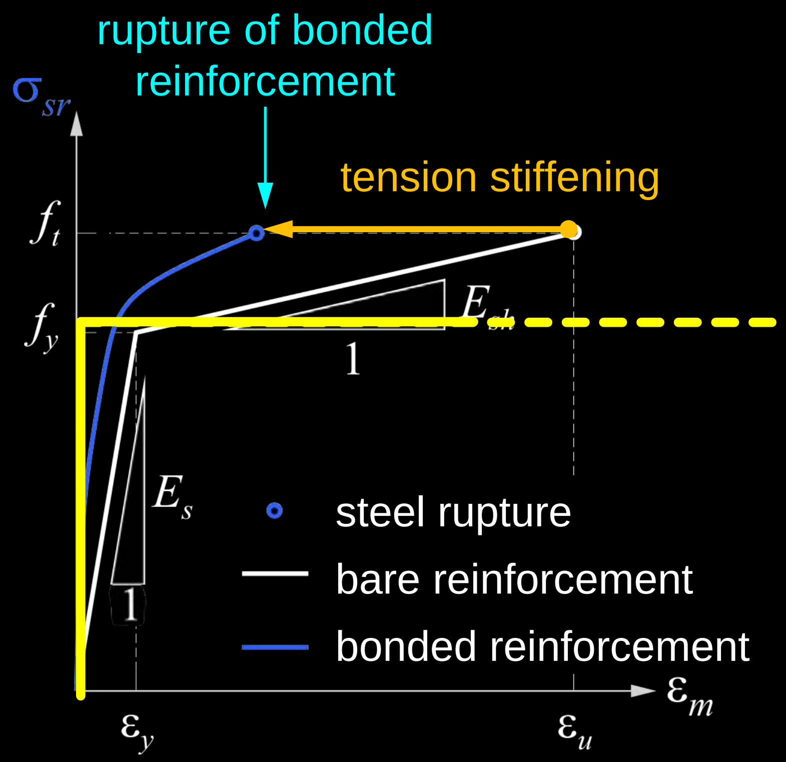
cautious value of
effective strength

Concrete in (indirect) tension

concrete in
tension
neglected

NB. Also due to unknown
stress state (initial restraint)

Reinforcing steel

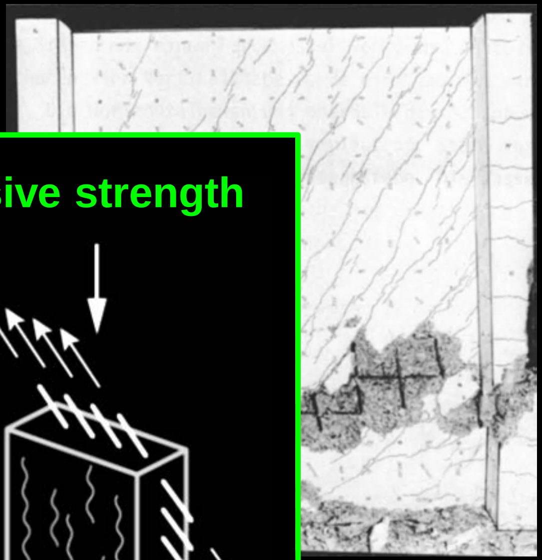


dowel action neglected
ductility deemed sufficient

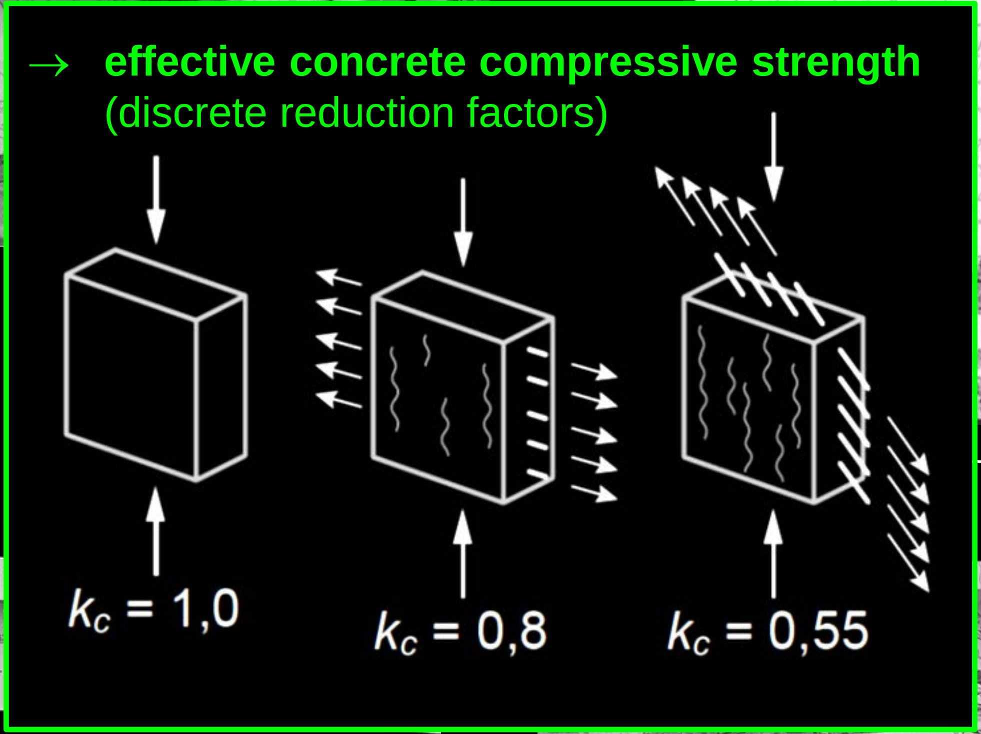
Plasticity-based models – Zürich school

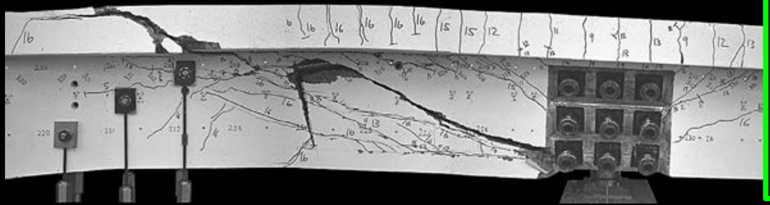
Limit analysis: Experimental validation

[Bachmann / Thürlimann 1965]

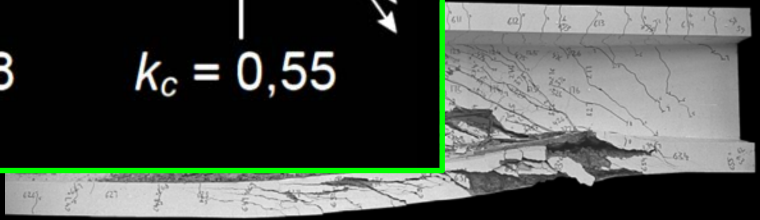


Thürlimann 1985]





[Stoffel / Marti 1995]



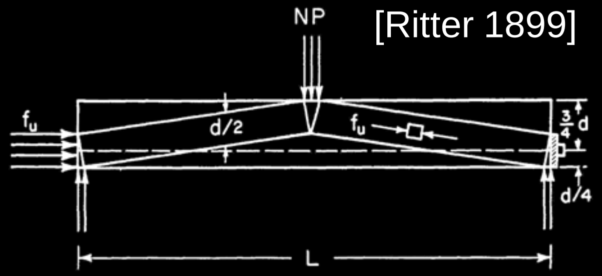
[Kaufmann / Marti 1995]

Bruno Thürlimann
1923-2008

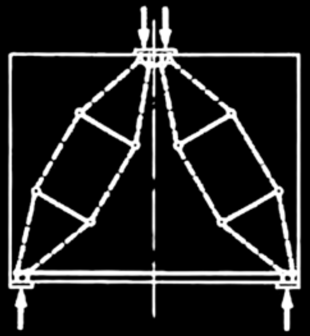
Peter Marti
*1949

and many others:
Paul Lampert,
Peter Müller,
Josef Grob,
Aurelio Muttoni,
…

[Ritter 1899]



[Drucker 1961]

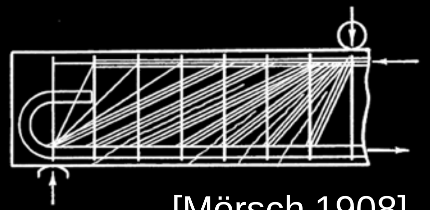


[Schlaich et al. 1987]

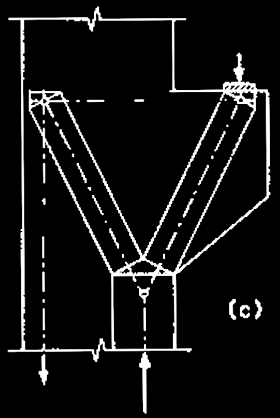
lower-bound theorem of limit analysis:
consistent mechanical framework for
truss models / strut-and-tie models as

(dis)continuous stress fields

Past **Present** **Future** ?



[Mörsch 1908]



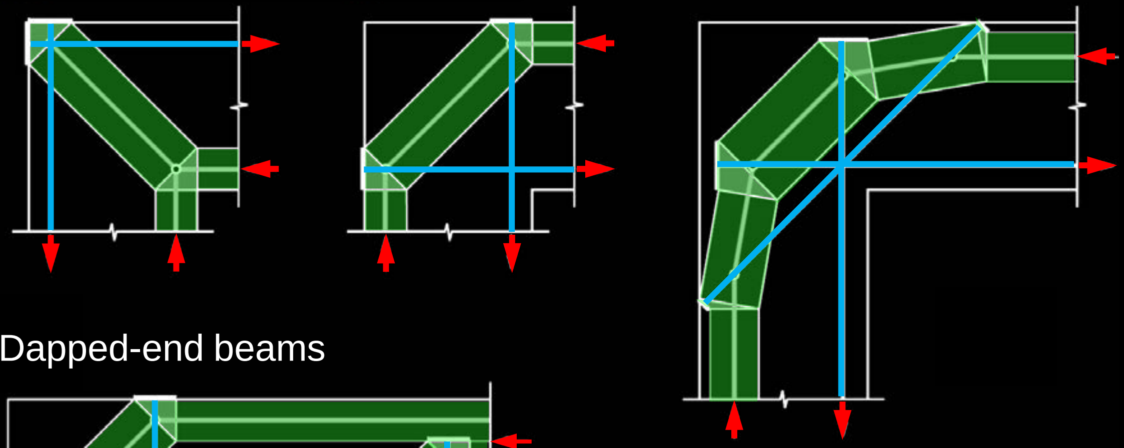
[Marti 1985]

conceptual measures to ensure ductility:
ensure reinforcement yielding at failure

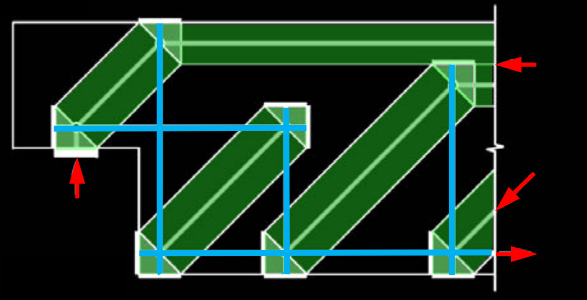
→ conservative value of «effective» concrete compressive strength

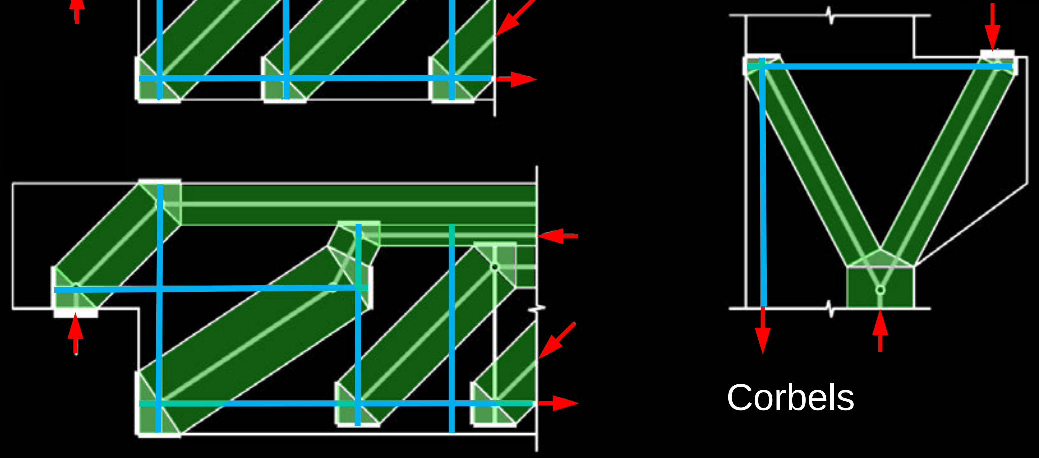
Discontinuous stress fields / STM

Frame corners

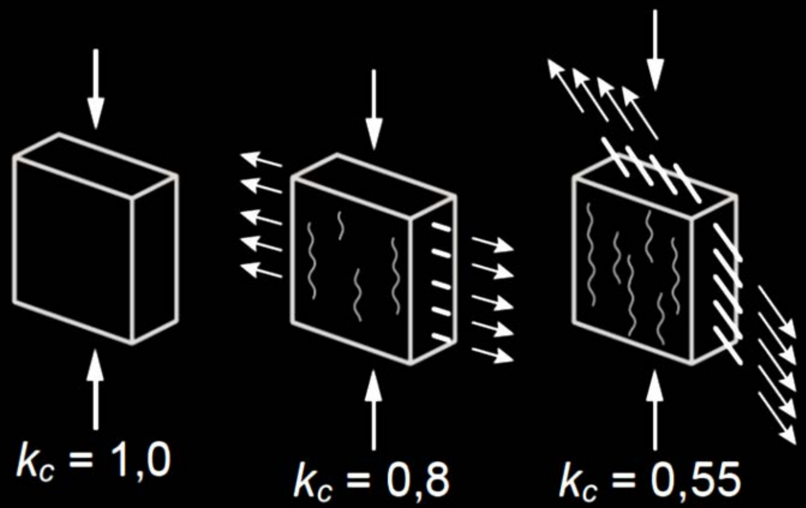


Dapped-end beams





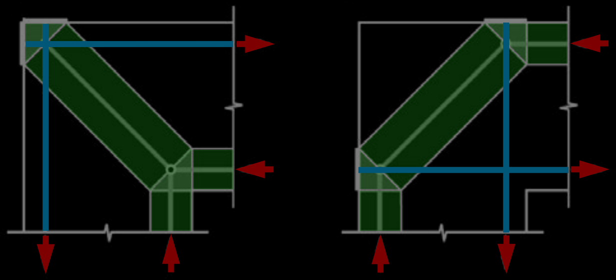
Corbels

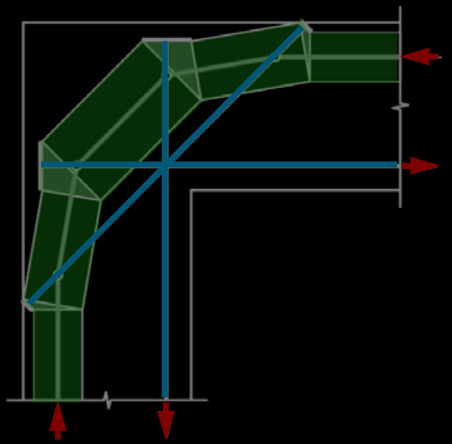


effective concrete compressive strength chosen by engineering judgement

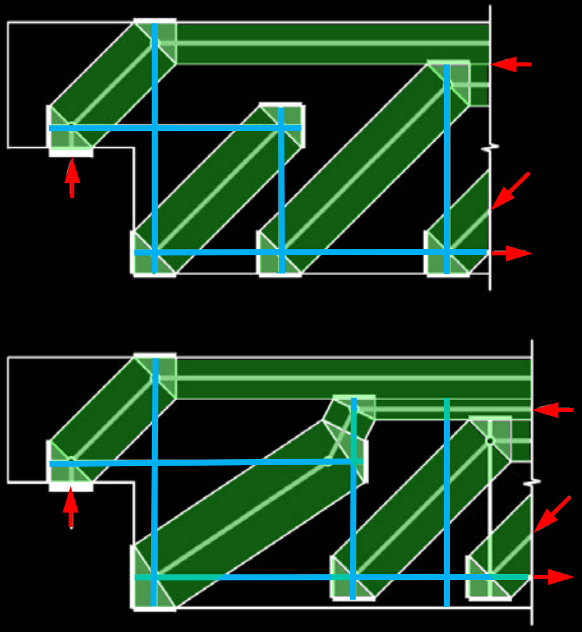
Discontinuous stress fields / STM

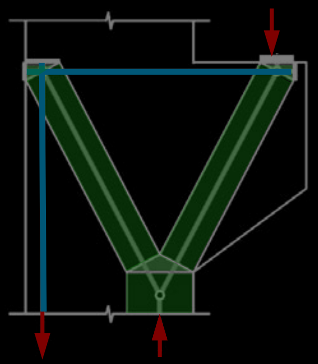
Frame corners





Dapped-end beams





Corbels

No unique solution for a given problem

→ designer tells the structure how to carry the load

Guidelines for stress field development:

- simplicity
- practicality (→ reinforcement layout)
- efficiency
- stiffness (minimise complementary energy)

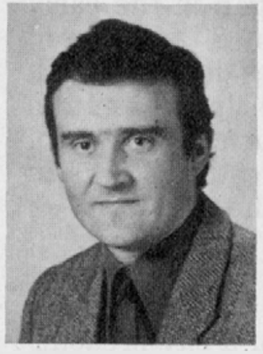
effective concrete compressive strength

chosen by engineering judgement

Discontinuous stress fields / STM

P. Marti, “Truss Models in Detailing”
Concrete International

1985



Recent advances in computer development indicate that there might be much better ways of using the unique capabilities of computers to assist reinforced concrete designers in the future. There is a considerable potential for applying interactive computer programs with graphical input and output that could replace the traditional drawing board and pocket calculator methods for developing truss models. Apart from ultimate strength considerations, such programs would allow us to investigate the deformations in the cracked state by taking appropriate truss member stiffnesses into account.

No unique solution for a given problem

→ designer tells the structure how to carry the load

Guidelines for stress field development:

- simplicity
- practicality (→ reinforcement layout)
- efficiency
- stiffness (minimise complementary energy)

Stress field development by hand

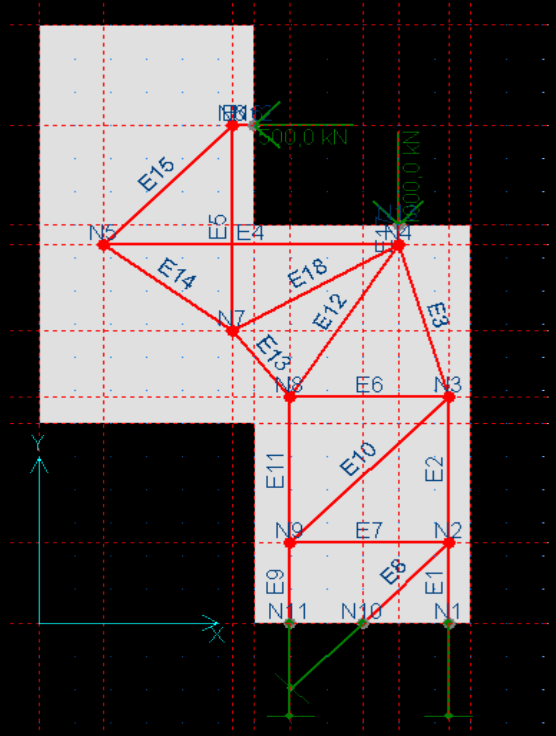
Limiting factors for widespread use in industry:

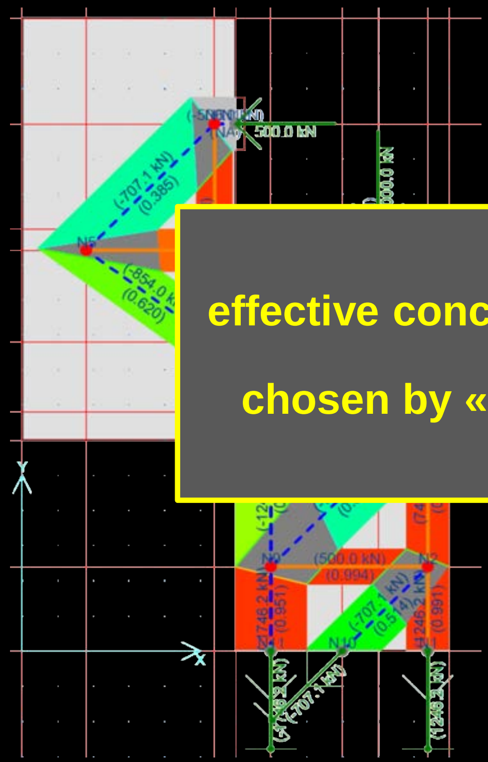
- requires engineering judgment
- time-consuming iterative adjustments
- limited use for SLS verifications

→ adequate in the 21st century?

Computer-aided discontinuous stress fields / STM

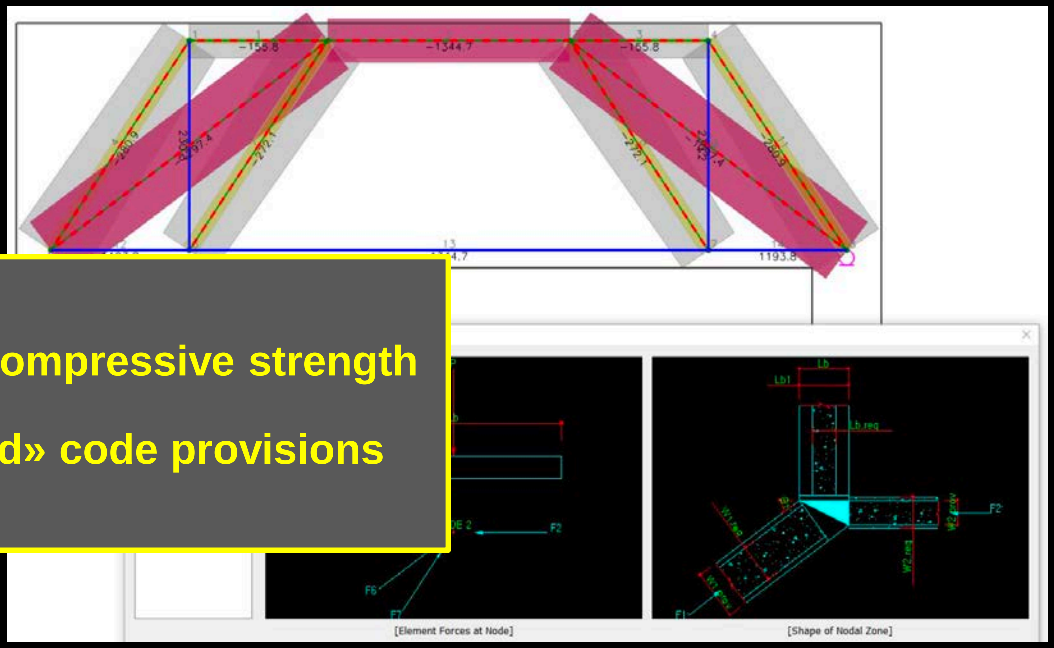
CAST (Tjhin & Kutchma, 2002)



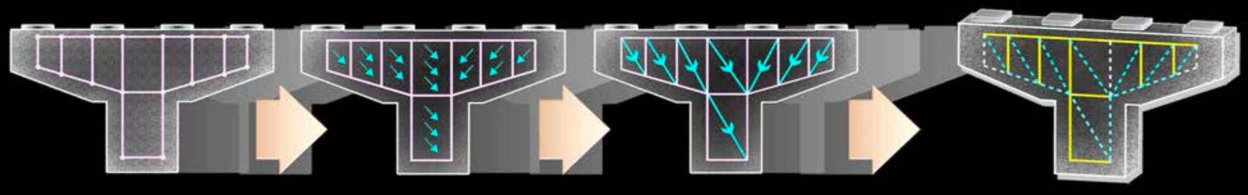


[Mata-Falcón & Sánchez-Sevilla, 2006]

AStrutTie 2020 (HanGil)



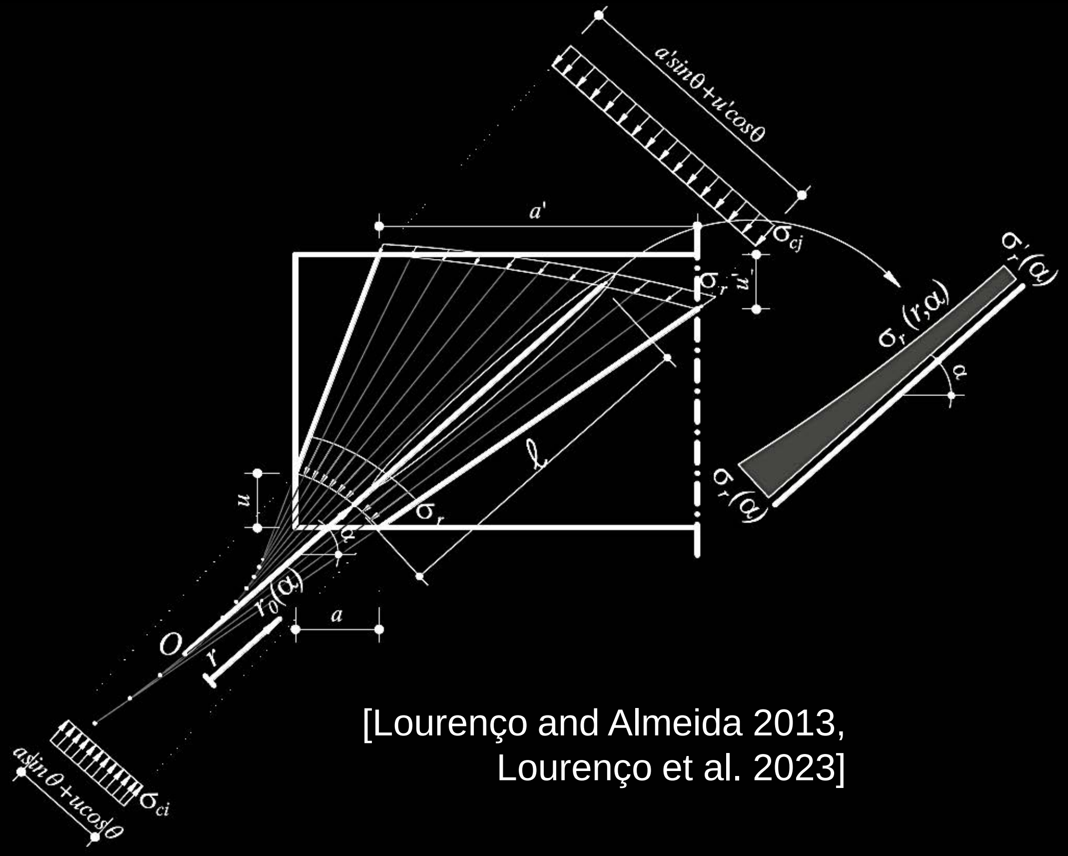
effective concrete compressive strength chosen by «refined» code provisions



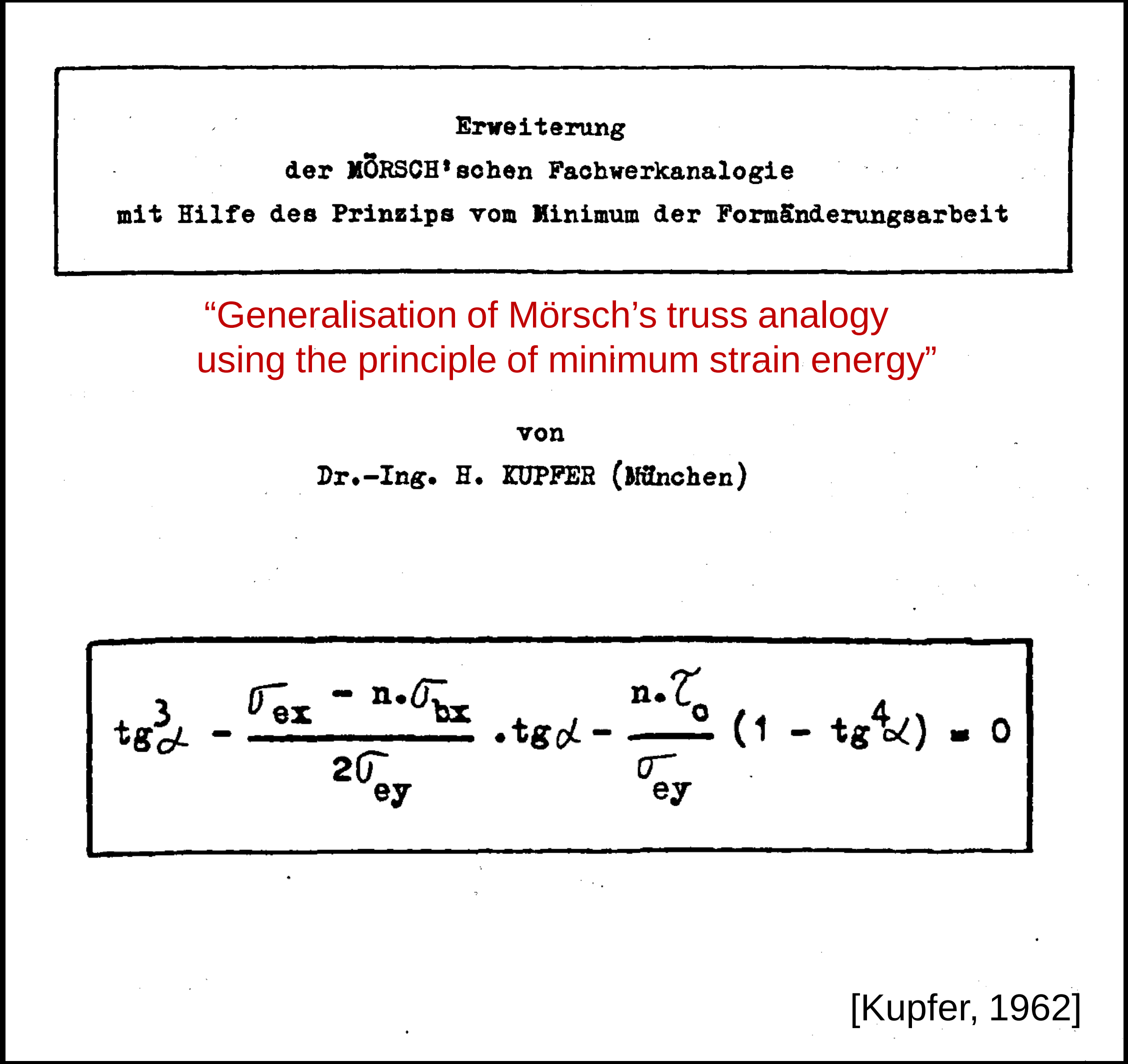
Computer-aided discontinuous stress fields / STM

Adaptive Stress Field Method

minimising complementary energy



[Lourenço and Almeida 2013,
Lourenço et al. 2023]



Erweiterung
der MÖRSCH'schen Fachwerkanalogie
mit Hilfe des Prinzips vom Minimum der Formänderungsarbeit

"Generalisation of Mörsch's truss analogy
using the principle of minimum strain energy"

von
Dr.-Ing. H. KUPFER (München)

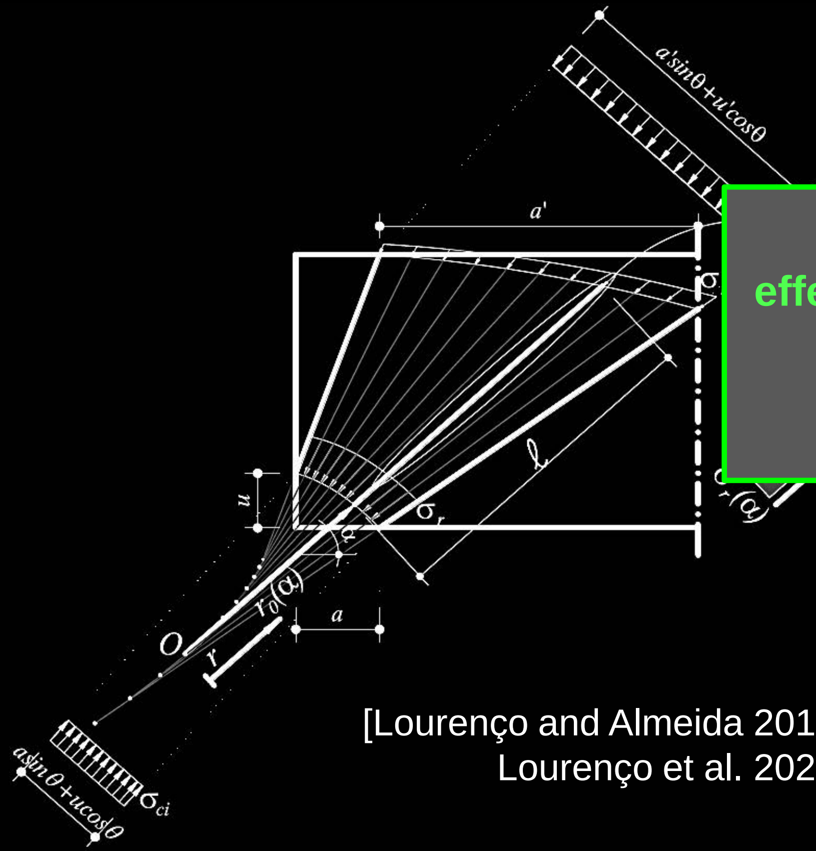
$$\mathrm{tg}^3\alpha - \frac{\sigma_{ex} - n \cdot \sigma_{bx}}{2\sigma_{ey}} \cdot \mathrm{tg}\,\alpha - \frac{n \cdot \tau_o}{\sigma_{ey}} \left(1 - \mathrm{tg}^4\alpha\right) = 0$$

[Kupfer, 1962]

Computer-aided discontinuous stress fields / STM

Adaptive Stress Field Method

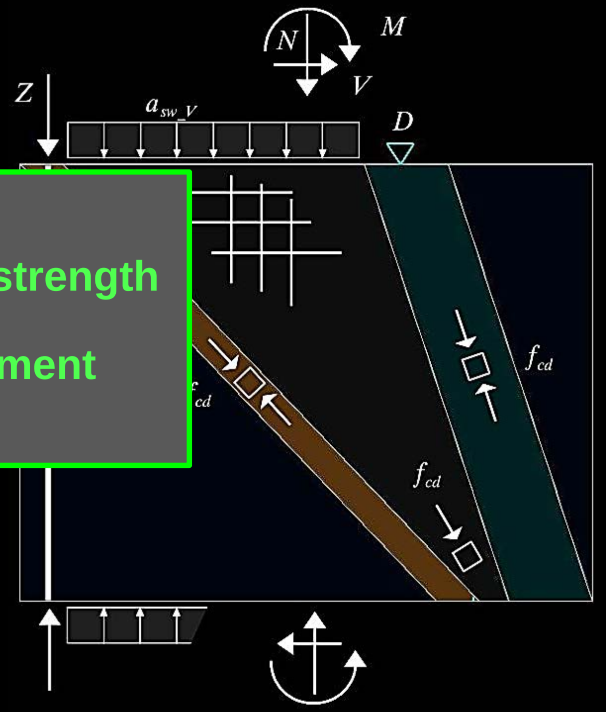
minimising complementary energy



[Lourenço and Almeida 2013,
Lourenço et al. 2023]

Parametric Stress Fields

adapted to activate distributed reinforcement

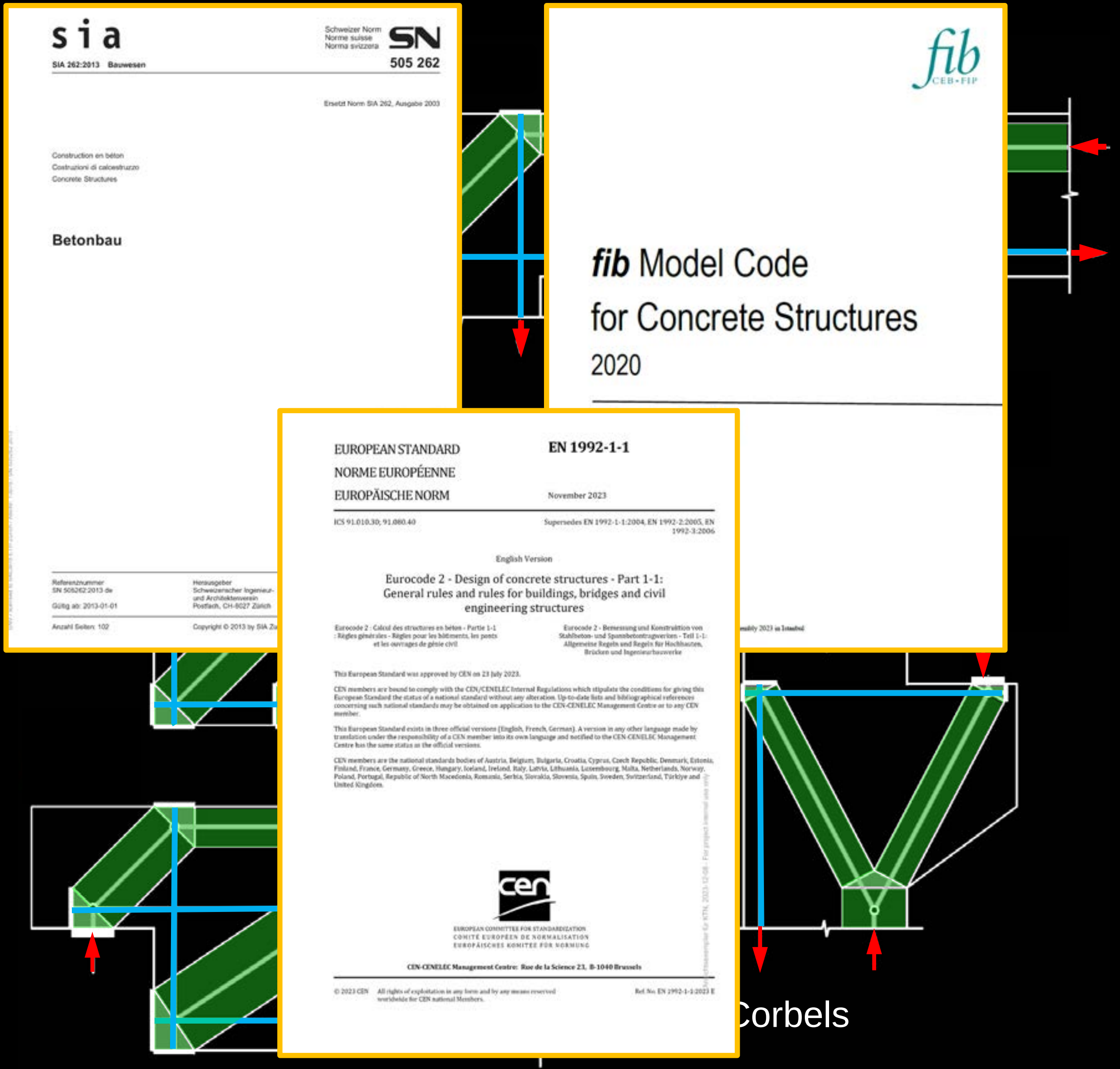


[Cubus AG, 2024]

effective concrete compressive strength

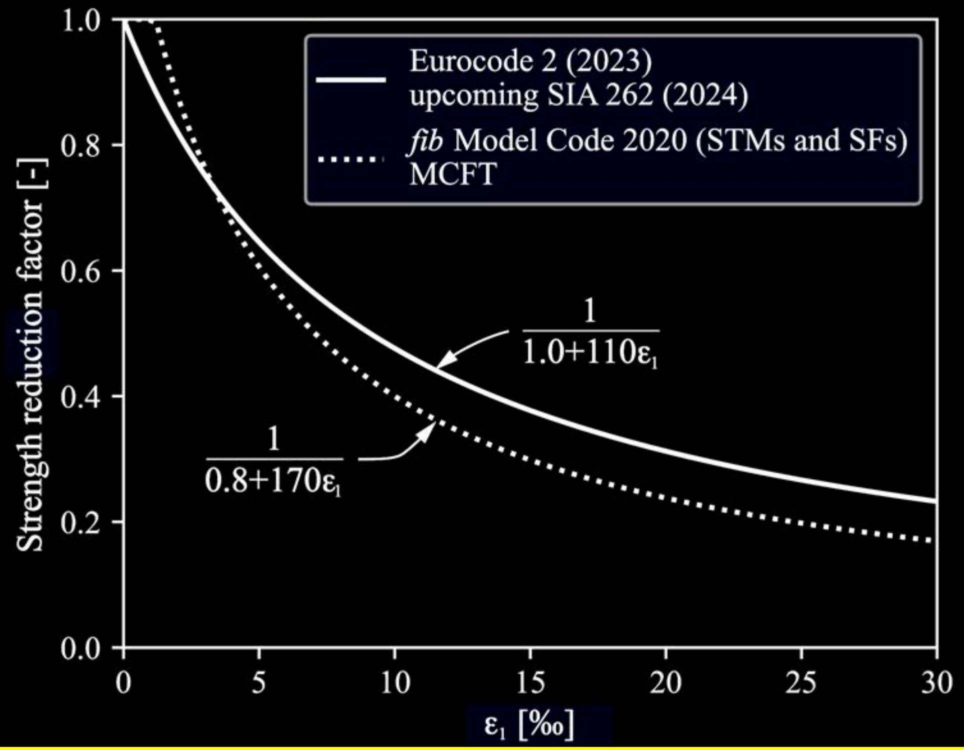
chosen by engineering judgement

Computer-aided discontinuous stress fields / STM

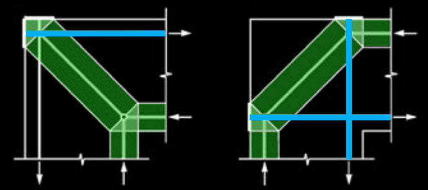


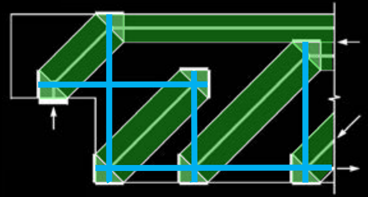
effective concrete compressive strength
depending on principal tensile strain

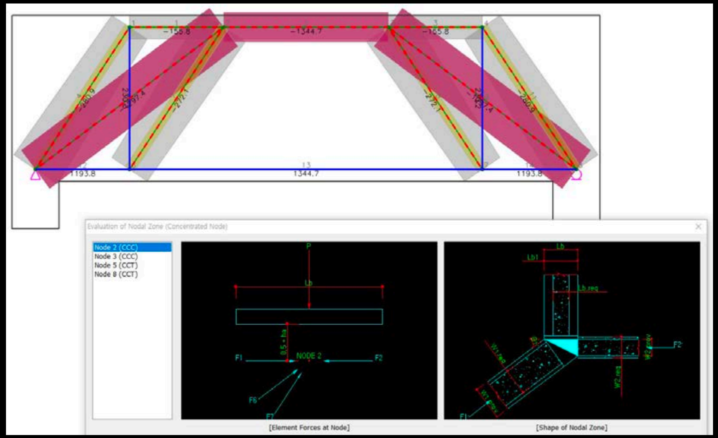
unknown in classic stress fields

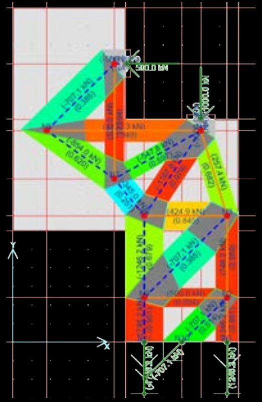


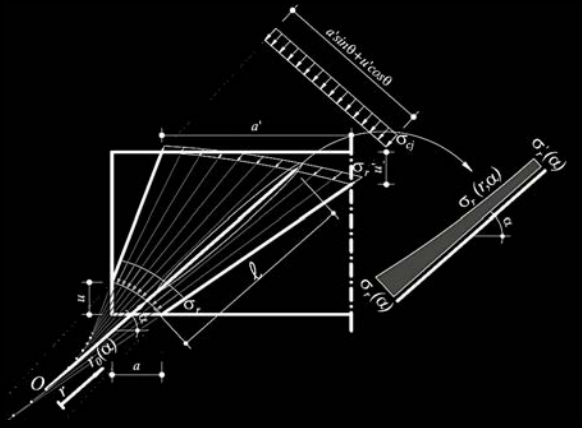
Computer-aided discontinuous stress fields / STM

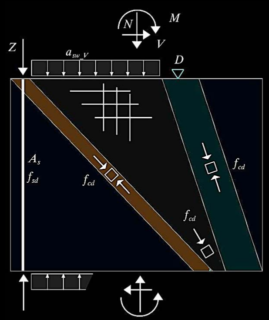






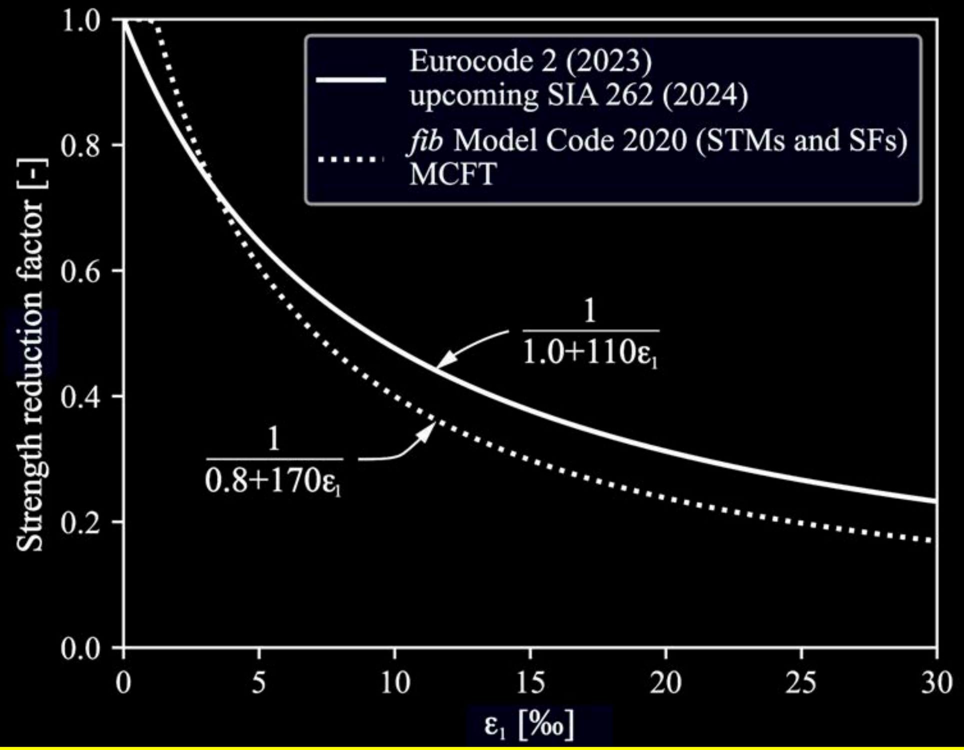




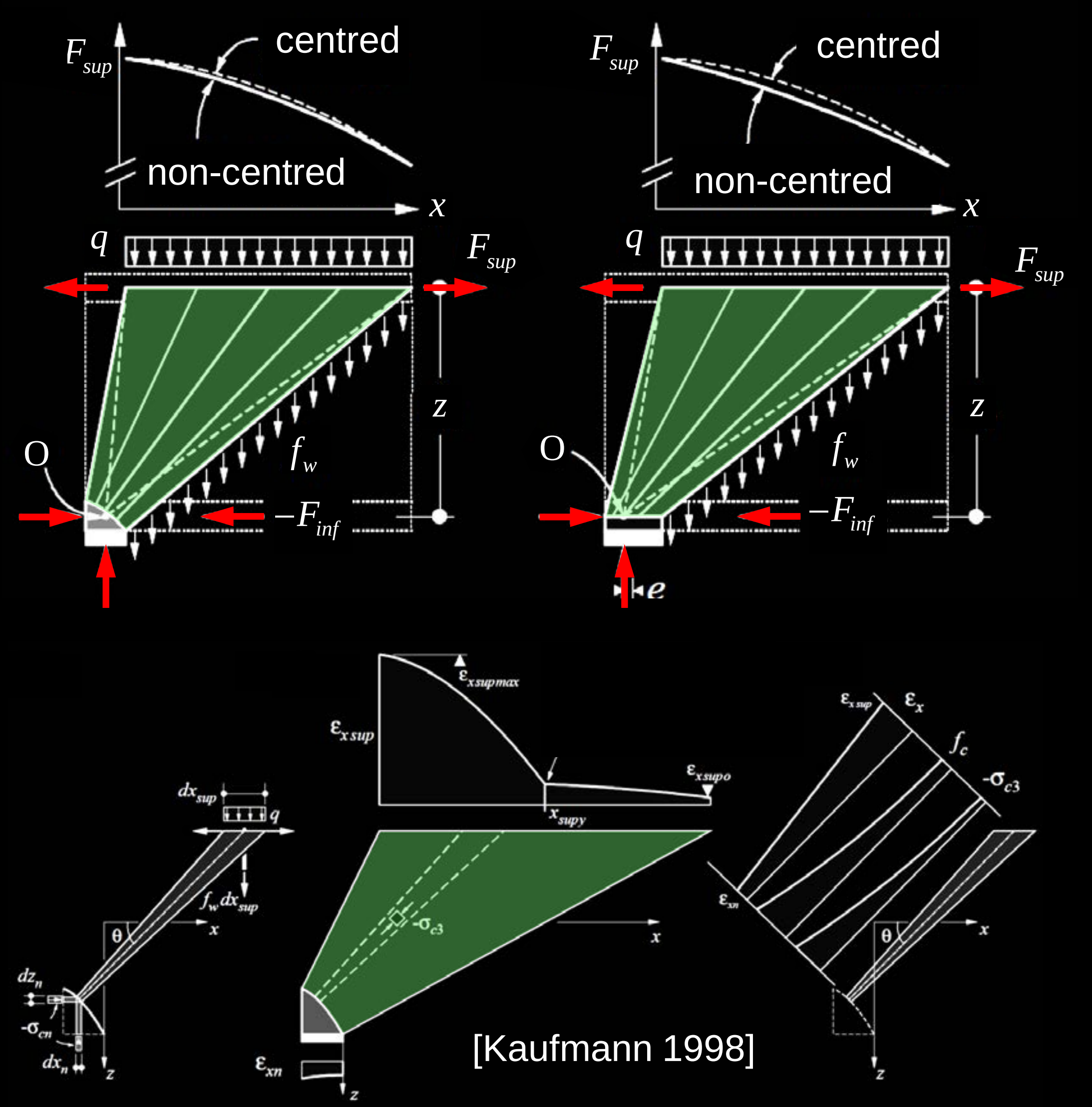


effective concrete compressive strength
depending on principal tensile strain

unknown in classic stress fields



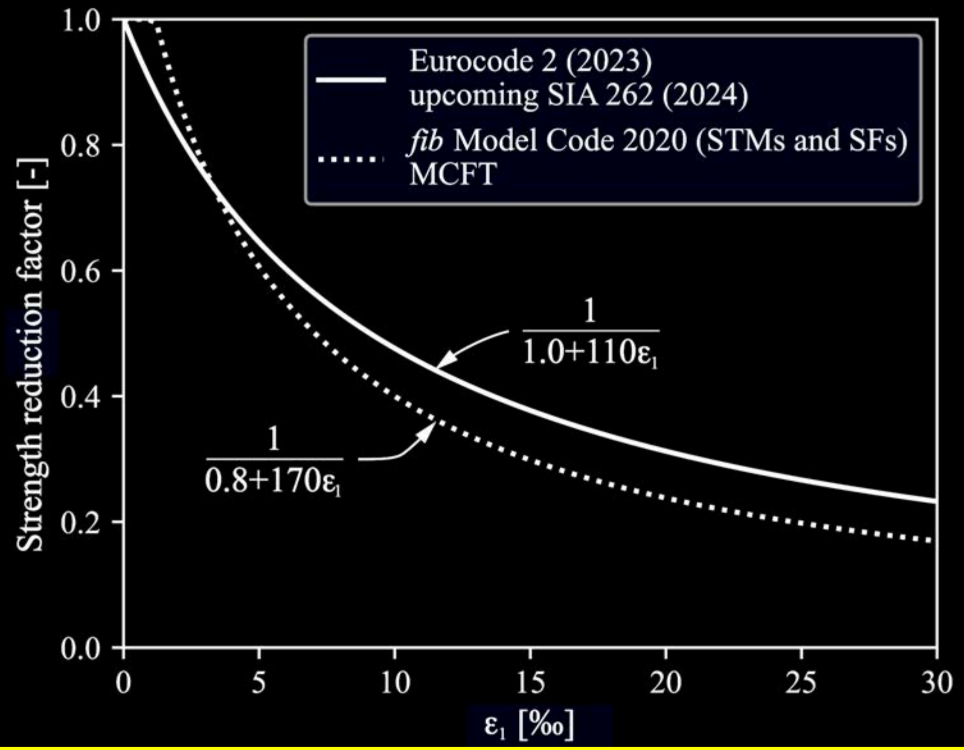
Computer-aided discontinuous stress fields / STM



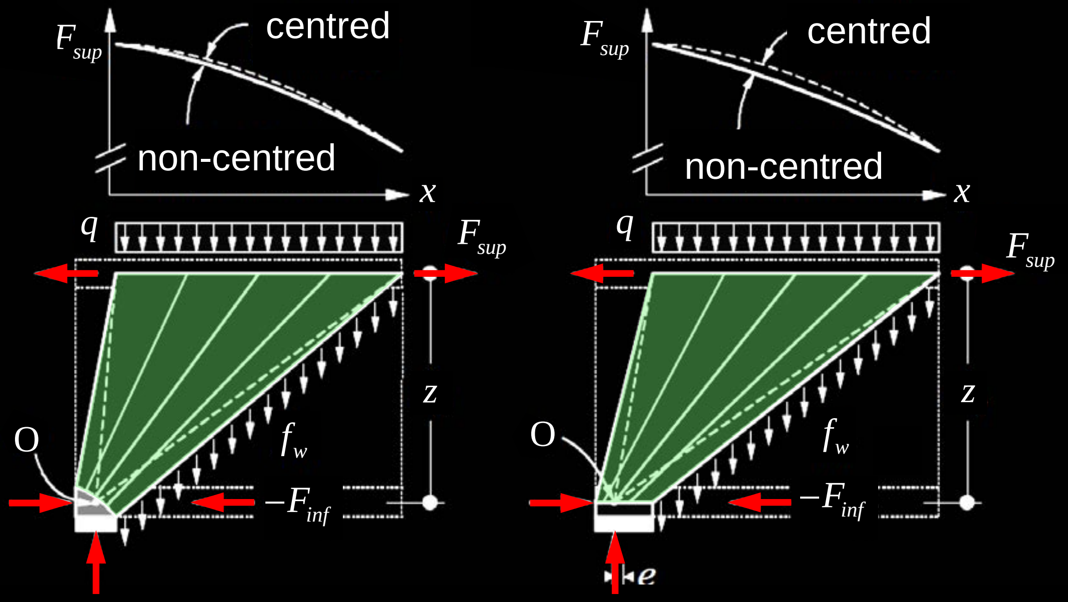
effective concrete compressive strength depending on principal tensile strain

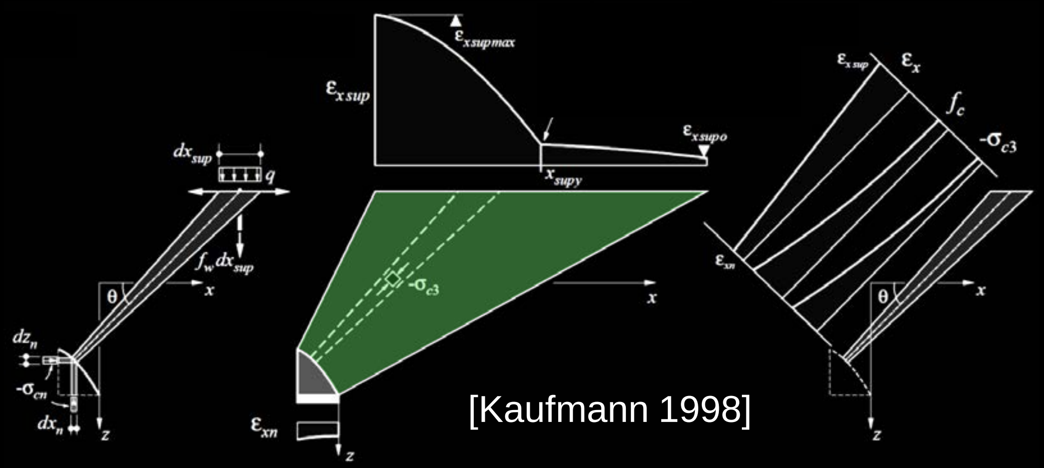
unknown in classic stress fields

directly applicable only for special cases



Computer-aided discontinuous stress fields / STM



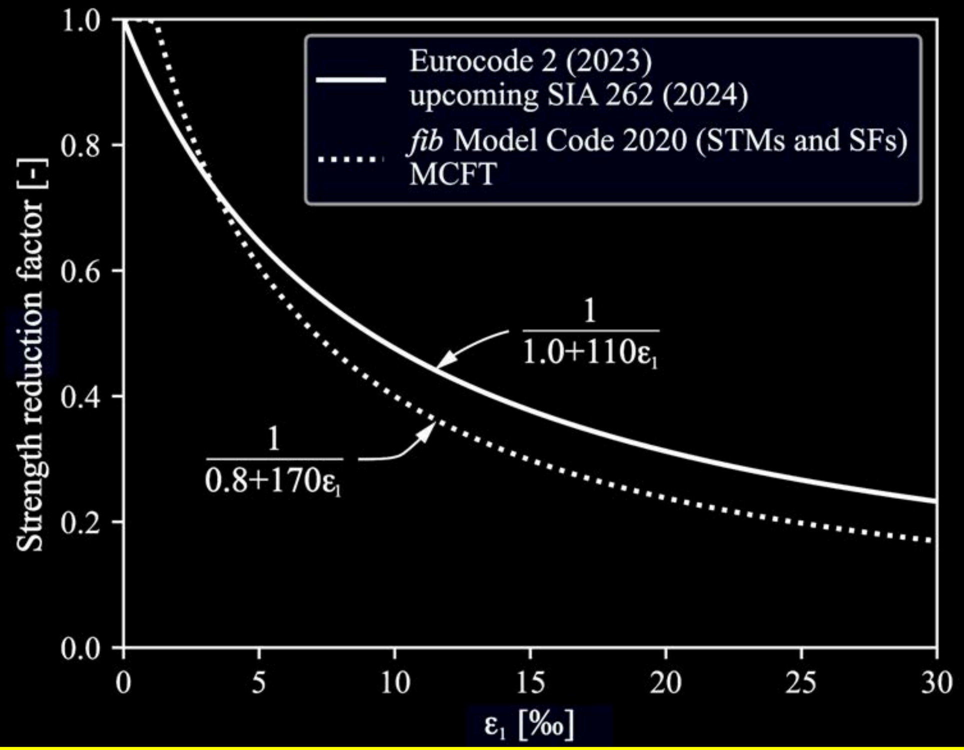


[Kaufmann 1998]

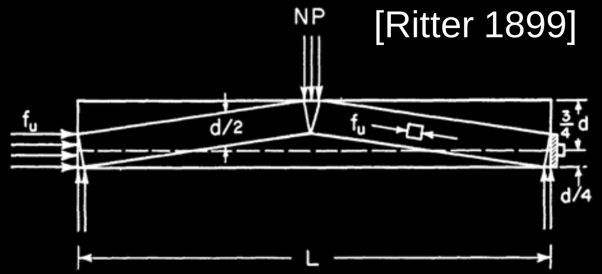
effective concrete compressive strength
depending on principal tensile strain

unknown in plasticity-based stress-fields

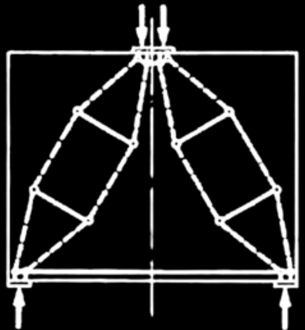
suitable for FE-based computational tools



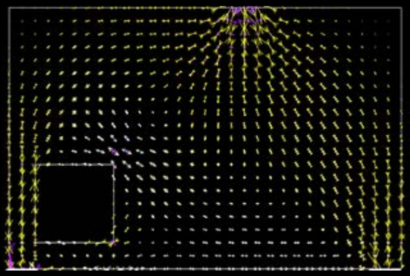
[Ritter 1899]



[Drucker 1961]



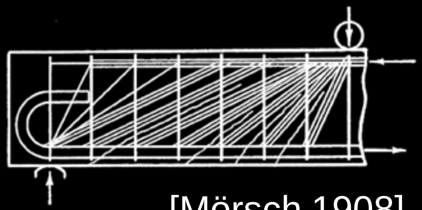
[Schlaich et al. 1987]



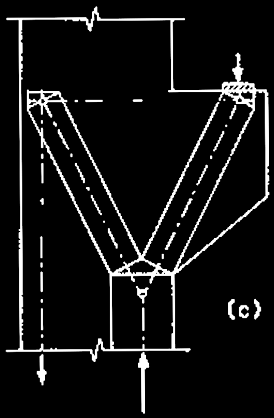
[Kaufmann et al., 2020]

- concrete in tension neglected
  → compatible with limit analysis
- concrete in compression: parabolic (with k_c)
- accounting for tension stiffening
  (Tension Chord Model)

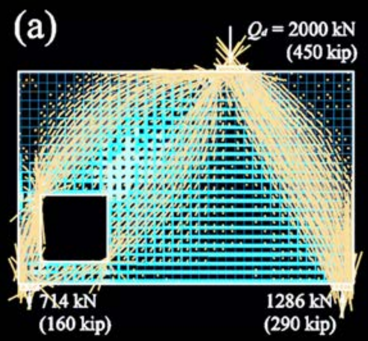
?



[Mörsch 1908]



[Marti 1985]

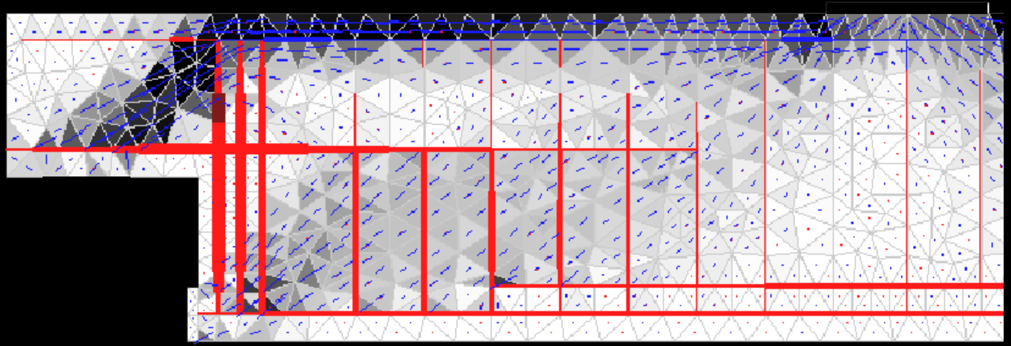


[Muttoni, Fernández Ruiz et al. 2015]

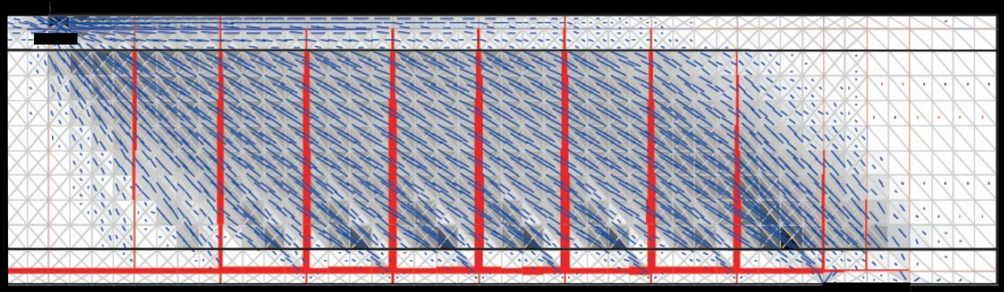
- concrete in tension neglected
  → compatible with limit analysis
- reinforcement and concrete in compression:
  linear elastic – perfectly plastic (with k_c)
- no account for tension stiffening

Lean RC-specific NLFE analyses (FE-based stress fields)

EPSF Elasto-Plastic Stress Fields
(Fernández Ruiz & Muttoni, 2007)

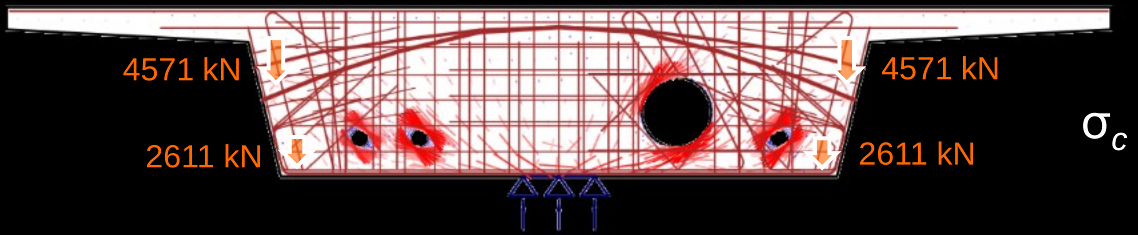


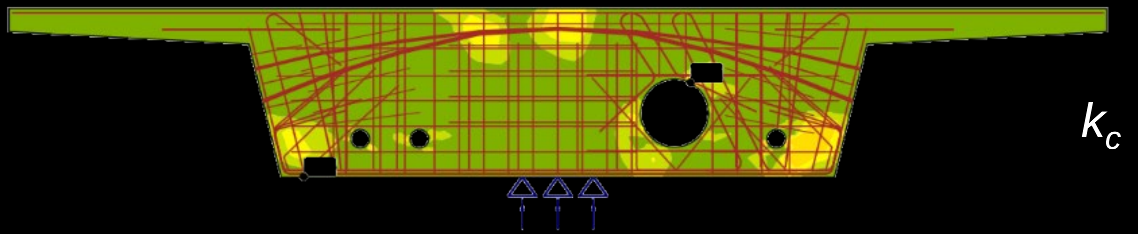
[Mata-Falcón, 2015]



[Mata-Falcón et al., 2014]

CSFM Compatible-Stress Field Method
(Kaufmann et al., 2020 → IdeaStatica Detail)



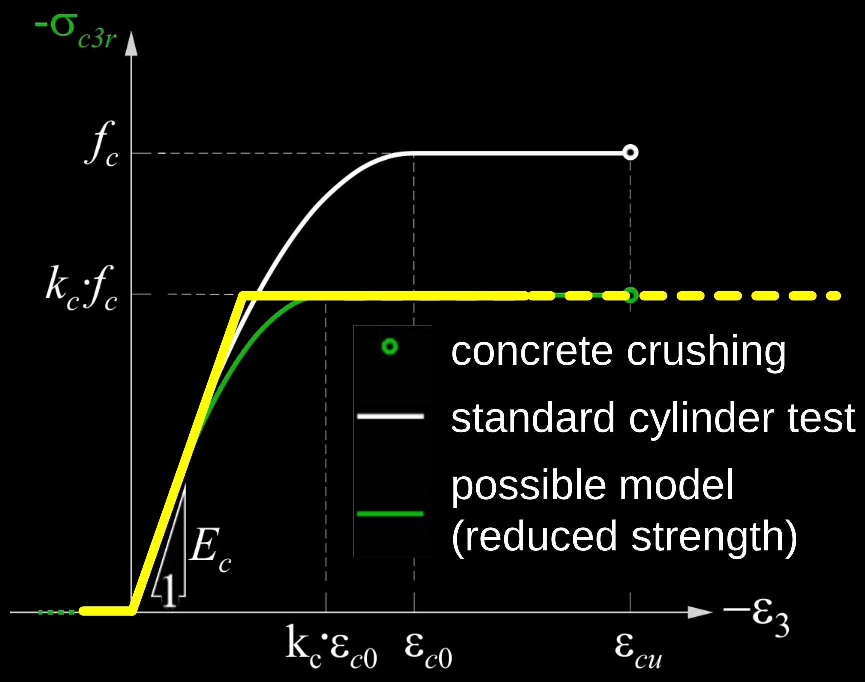


Development EPSF → CSFM:

- CSFM = user-friendly (commercial) software
- CSFM accounts for tension stiffening and anchorage
→ explicit check of deformation capacity
→ more realistic stiffness, suitable for SLS checks

Lean RC-specific NLFE analyses (FE-based stress fields)

Concrete in compression



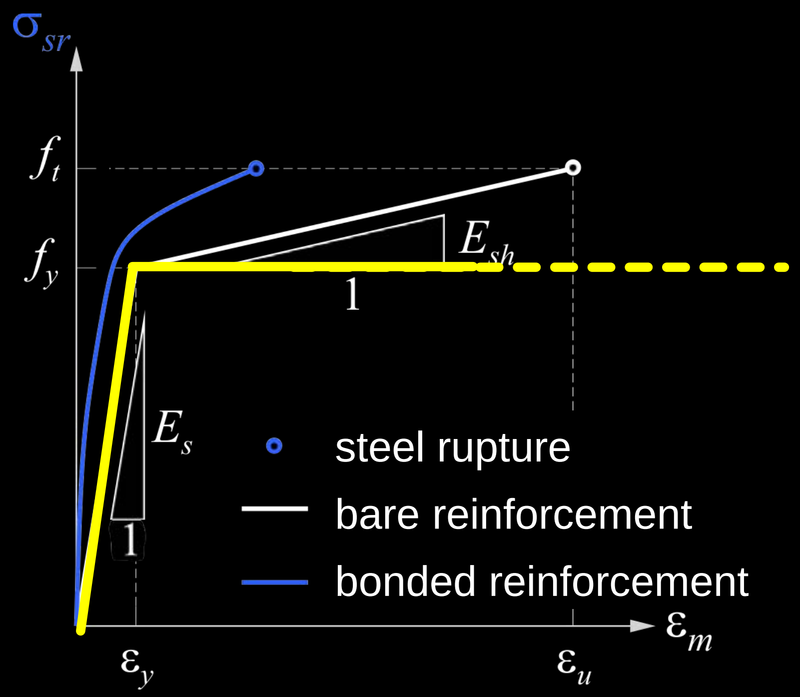
EPSF: bilinear, compression softening according to design codes

Concrete in (indirect) tension

concrete in tension neglected

NB. Also due to unknown stress state (initial restraint)

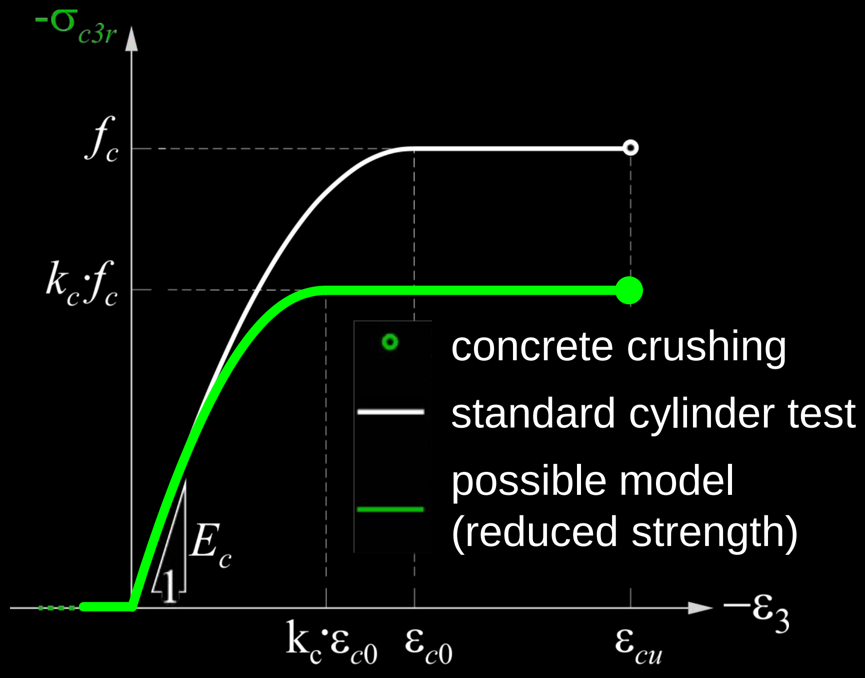
Reinforcing steel



EPSF: dowel action neglected ductility deemed sufficient

Lean RC-specific NLFE analyses (FE-based stress fields)

Concrete in compression



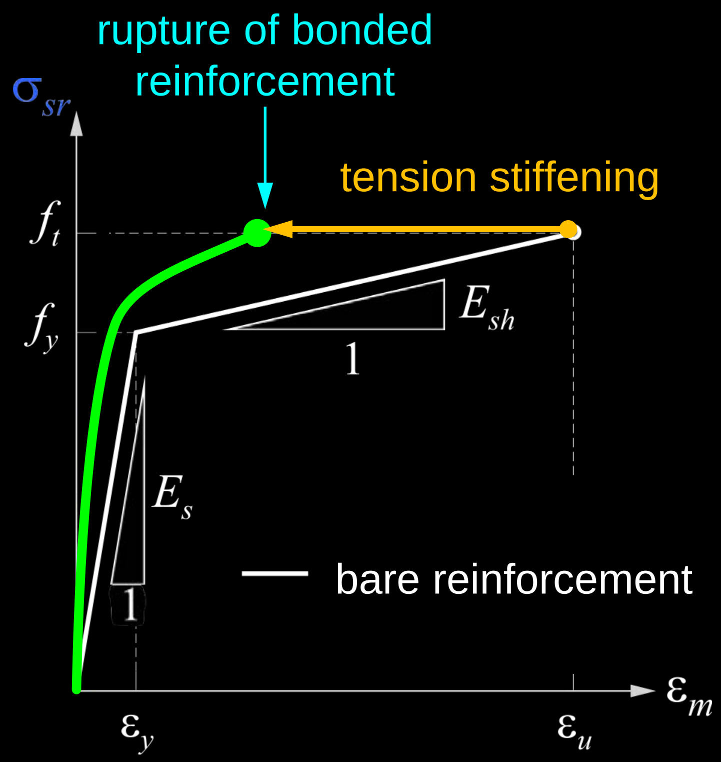
CSFM: parabolic, compression softening according to design codes

Concrete in (indirect) tension

concrete in tension neglected

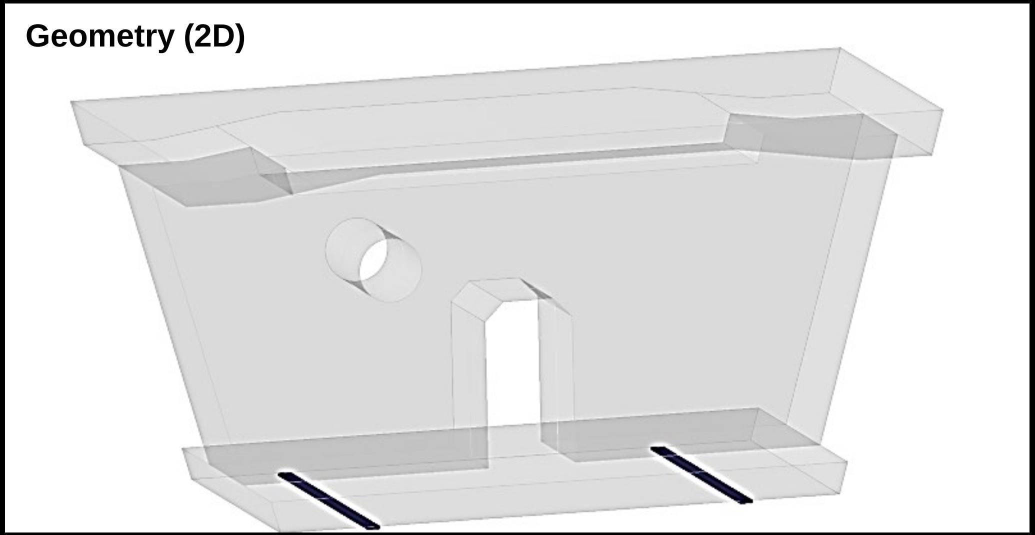
NB. Also due to unknown stress state (initial restraint)

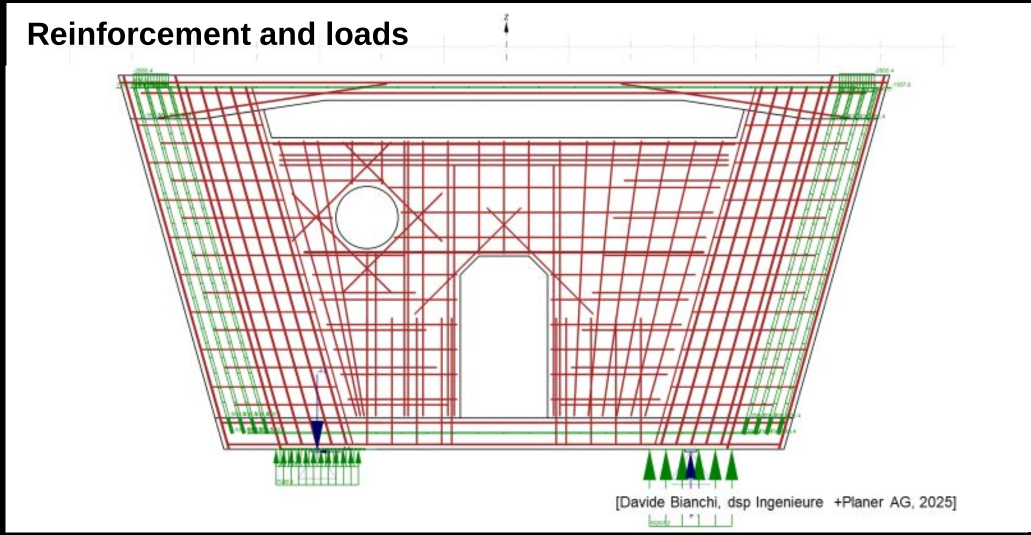
Reinforcing steel

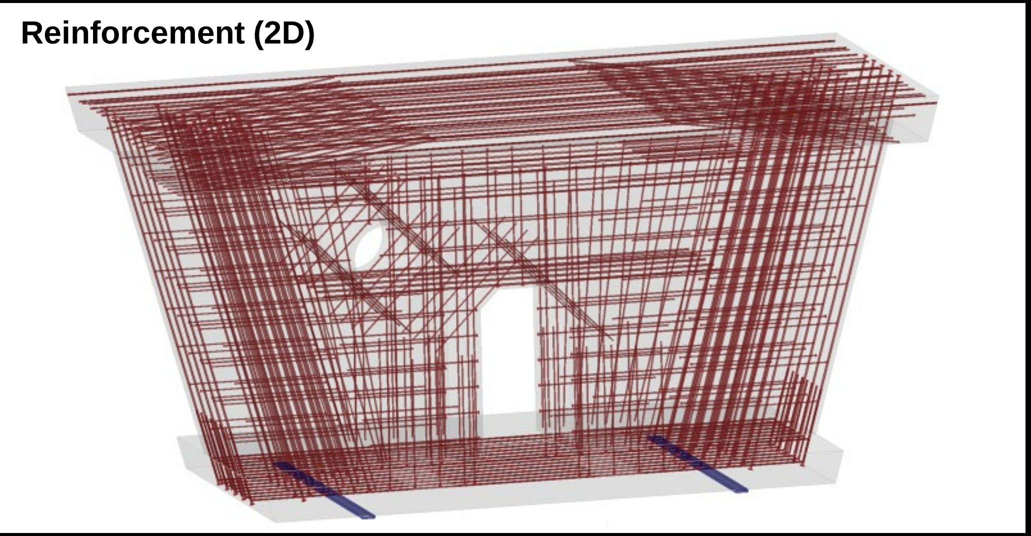


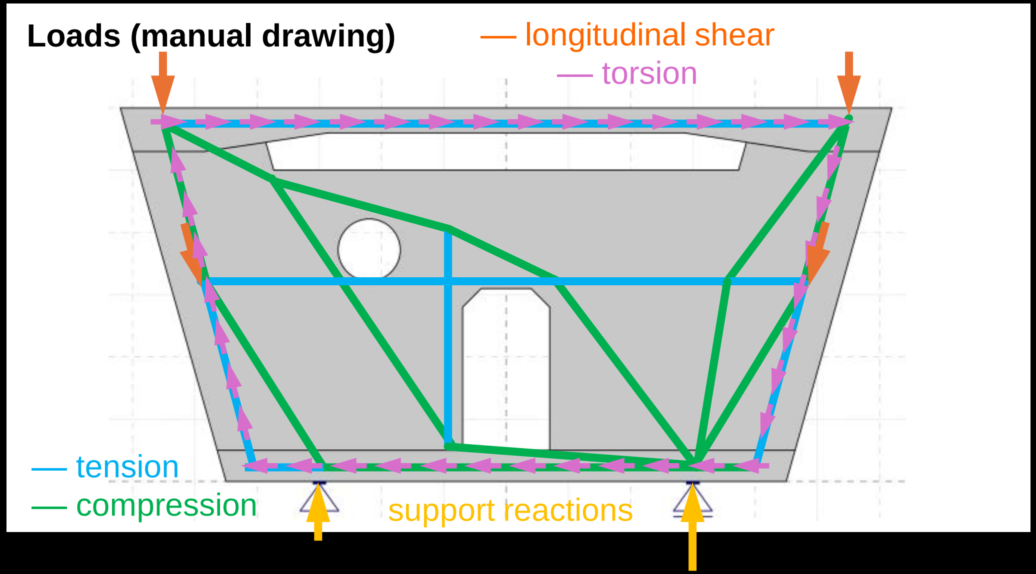
CSFM: dowel action neglected ductility explicitly checked

Lean RC-specific NLFE analyses – diaphragm example





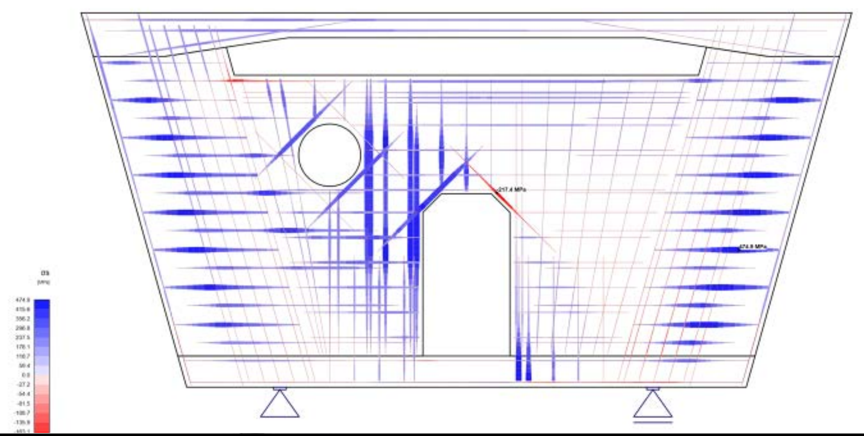




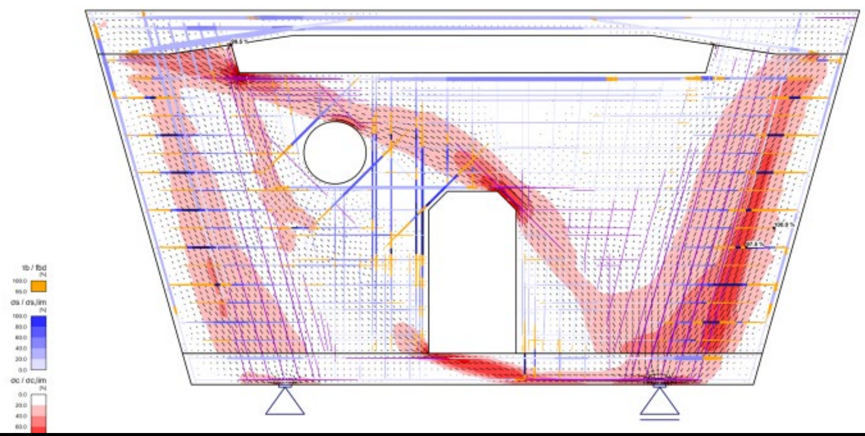
[Davide Bianchi, dsp Ingenieure + Planer AG, 2025]

Lean RC-specific NLFE analyses – diaphragm example

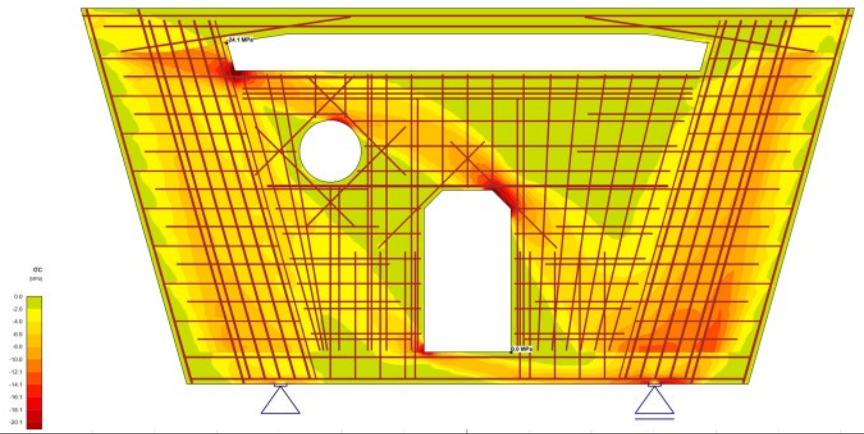
reinforcement stresses (at fictitious cracks)



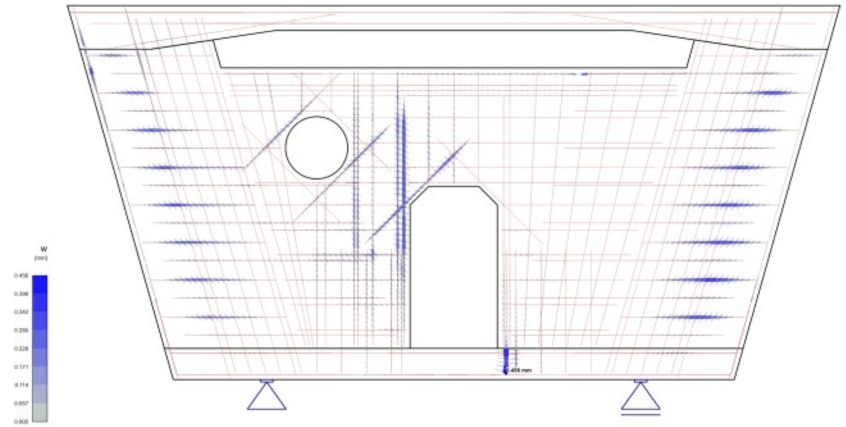
Utilisation (concrete, reinforcement, anchorage)



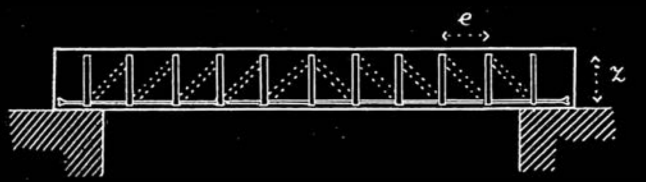
concrete stresses



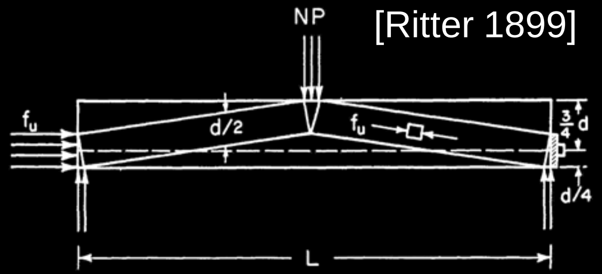
Crack widths



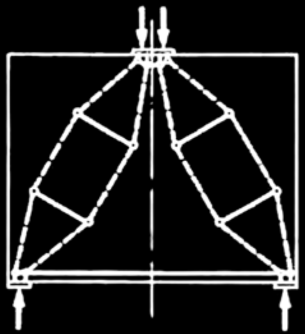
[Davide Bianchi, dsp Ingenieure + Planer AG, 2025]



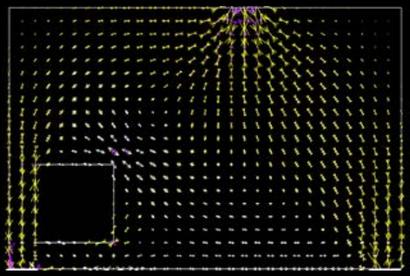
[Ritter 1899]



[Drucker 1961]

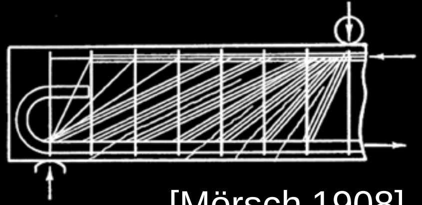


[Schlaich et al. 1987]

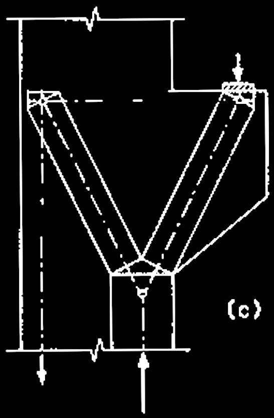


[Kaufmann et al., 2020]

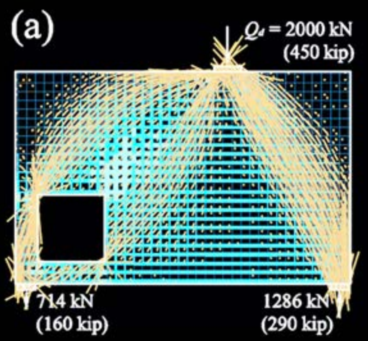
Past **Present** **Future** ?



[Mörsch 1908]

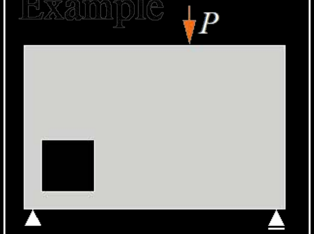
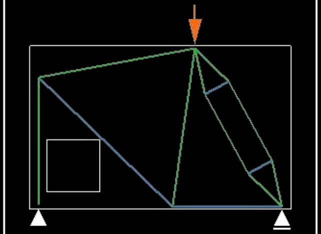
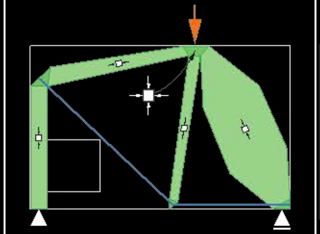
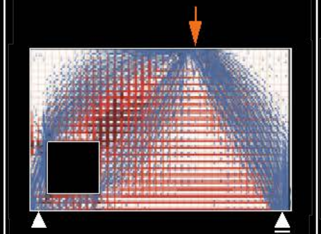
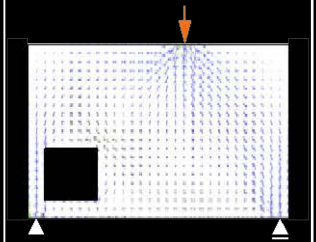


[Marti 1985]



[Muttoni et al. 2015]

Levels of approximation – 2D Models

Levels of approximation	LoA I	LoA II	LoA III	LoA IV
Example 			 [Muttoni et al. 2015]	
Material behaviour	rigid-perfectly plastic	rigid-perfectly plastic	linear elastic–perfectly plastic	nonlinear
Compatibility	No	No	Yes	Yes
Main application	Design (ULS)	Design (ULS)	Assessment (ULS)	Design (ULS/SLS) Assessment (ULS)
Application	Manually, partly automated [35], [41]	Manually, partly automated [38]	FEA	FEA
Model examples	Strut-and-tie models	Stress fields	Elasto-plastic stress fields	Cracked membrane models, compatible stress-field method

LoA I–II: STM, partly automated

- suitable for design
- interpretability
- engineering judgment required
- time-consuming
- limited use in assessment
- limited use for SLS verifications

LoA III–IV: lean RC-specific NLFE-analyses

- limited use for design
- lack of interpretability
- automated
- time-efficient
- suitable for assessment
- suitable for SLS verifications

3D?

Future development of stress fields / STM

General goals

Keep strut-and-tie models and stress fields alive by implementation in modern design tools …

… combining the advantages of manual and automated methods: TRANSPARENCY **AND** EFFICIENCY

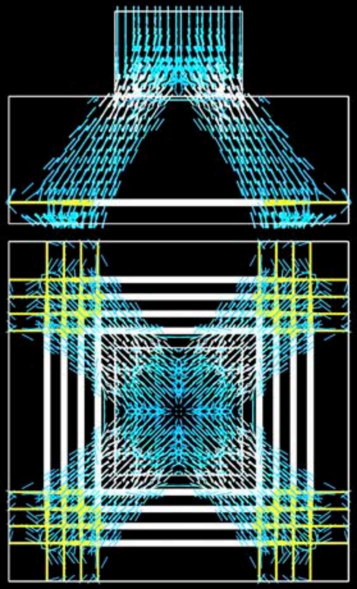
3D Problems

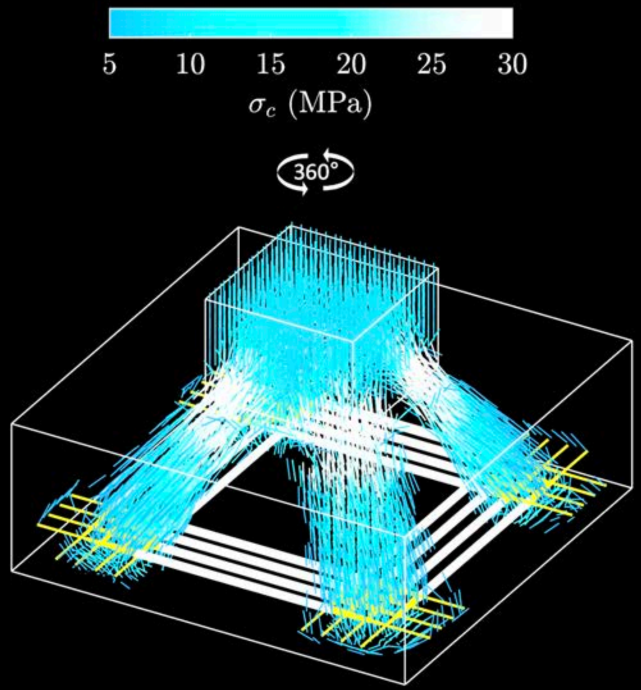
Improve the understanding of 3D nodal zones and stress fields

Develop a user-friendly, automated tool to develop reliable 3D strut-and-tie models and stress fields

Lean RC-specific NLFE analyses (FE-based stress fields) in 3D

FESCA Elasto-Plastic Stress Field in 3D



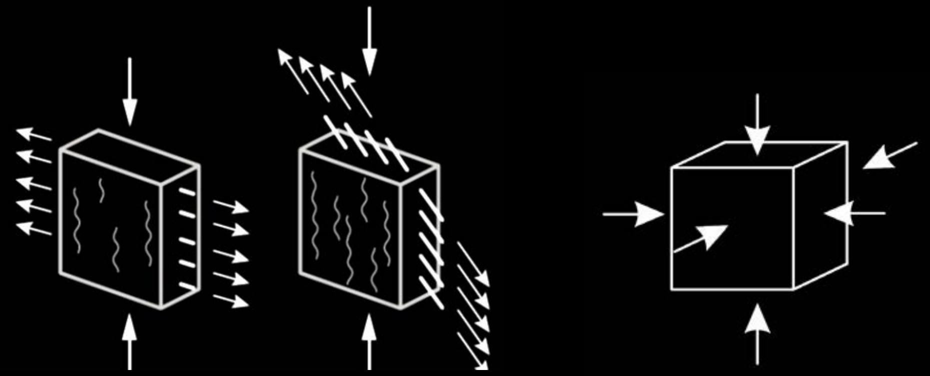


[Meléndez 2017]

Main challenges:

1. Realistic failure criterion of concrete

- compression softening in 3D
  vs. triaxial compression / confinement?



2. Maintaining simplicity and transparency

- avoid non-standard material parameters
- do not sacrifice mechanical consistency
  for the sake of numerical stability

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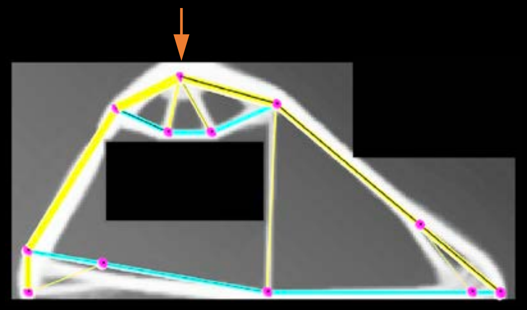
2D Problems

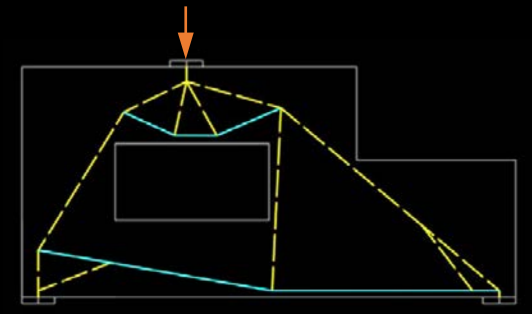
Design: develop a user-friendly end-to-end computational tool with strut-and-tie models and stress fields (e.g. define reinforcement with an automated strut-and-tie model, validate with CSFM)

Generative design methods for STM

Structural optimisation

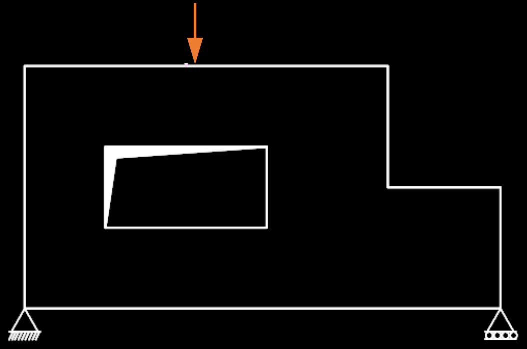
Continuum topology optimisation

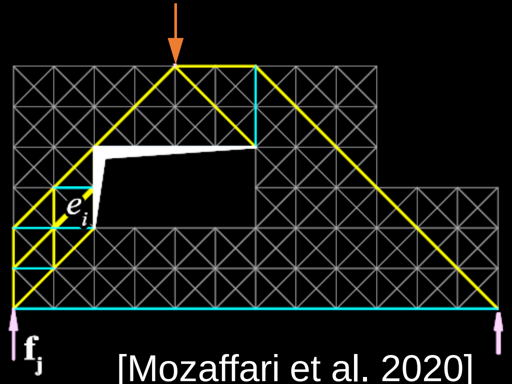




[Xia et al. 2020]

Discrete layout optimisation





[Mozaffari et al. 2020]

Continuum topology optimisation

- Material redistribution in a 2D or 3D continuum (with RC-specific material behaviour)
- Optimisation under certain constraints (e.g. stresses) and objectives (e.g. mass, stiffness)
- Truss extraction and checking of strut dimensions requires post-processing

Discrete layout optimisation

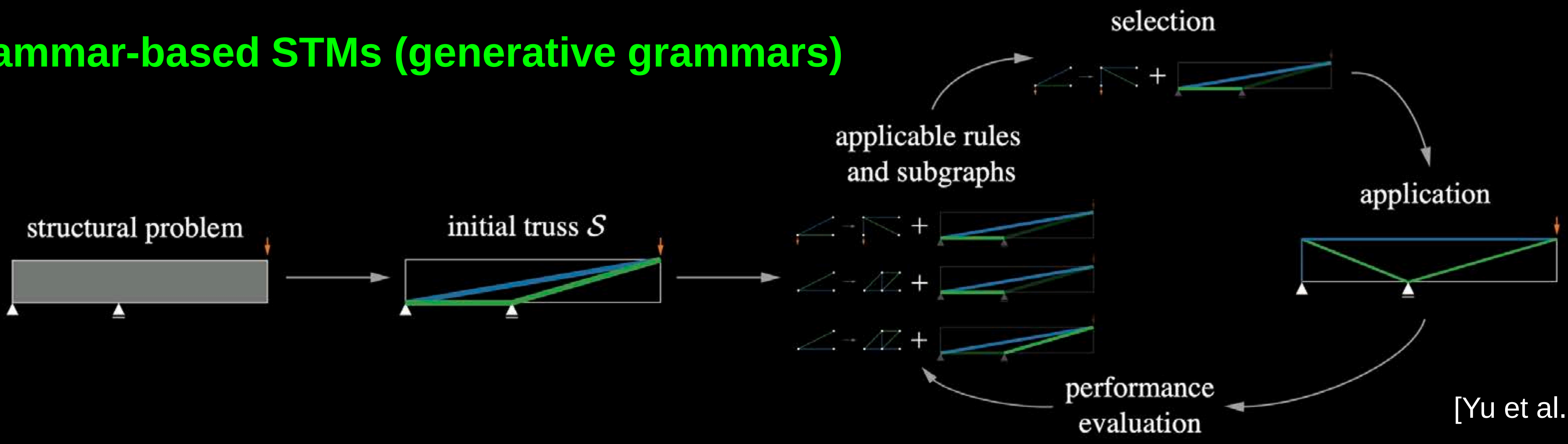
- Ground structure = connection of nodes in a grid-like pattern inside the concrete geometry
- Optimisation under certain geometrical and structural constraints and objectives (e.g. mass, stiffness)
- Checking of strut dimensions in post-processing

Truss-continuum topology optimisation

- combines both methods

Generative design methods for STM

Grammar-based STMs (generative grammars)



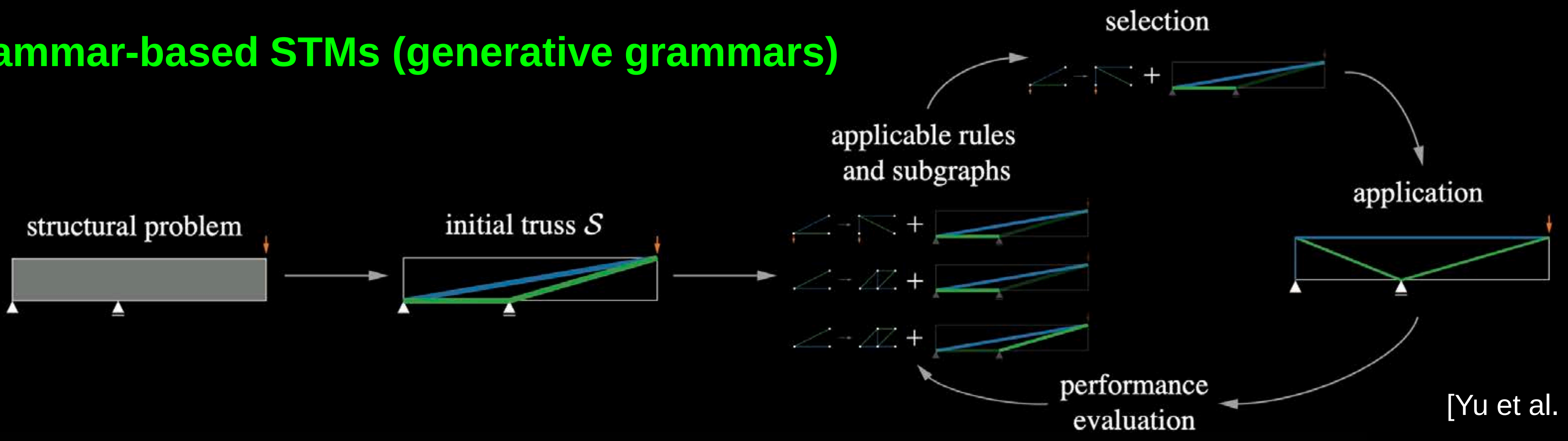
[Yu et al. 2024]

- Set of topological rules how an initial truss can evolve, preserving equilibrium
- Rules derived from constraints (validity) and requirements (suitability), allowing for engineering judgement
- Alternatives for application:
  … user-guided STM generation ("co-pilot")
  … automated STM generation with optimisation (e.g. mass or reinforcement complexity) for simpler cases.

NB. Ongoing research – see also https://concrete.ethz.ch/blog/grammars-for-strut-and-tie-models-explained/

Generative design methods for STM

Grammar-based STMs (generative grammars)



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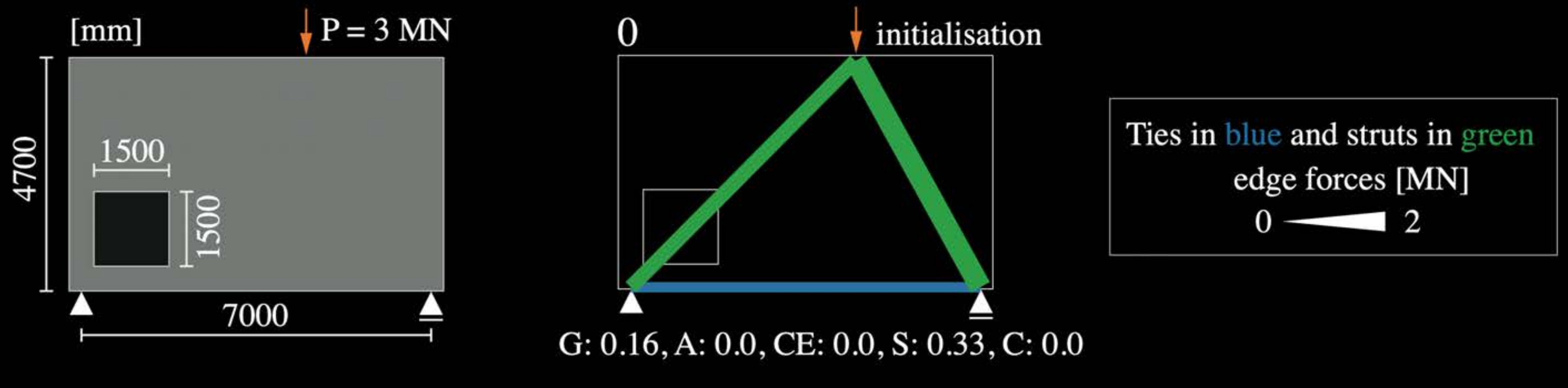

Metrics (this example):

Validity
- G geometry
- A strut angles
- CE intersections

Suitability / performance
- S steel consumption
- C reinforcement complexity

Generative design methods for STM

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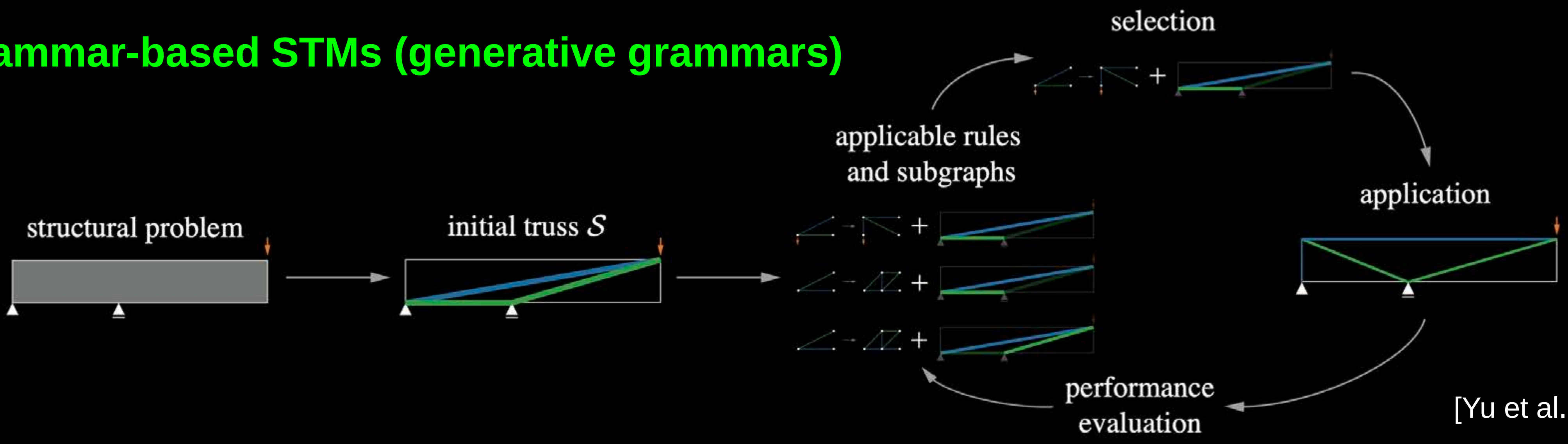
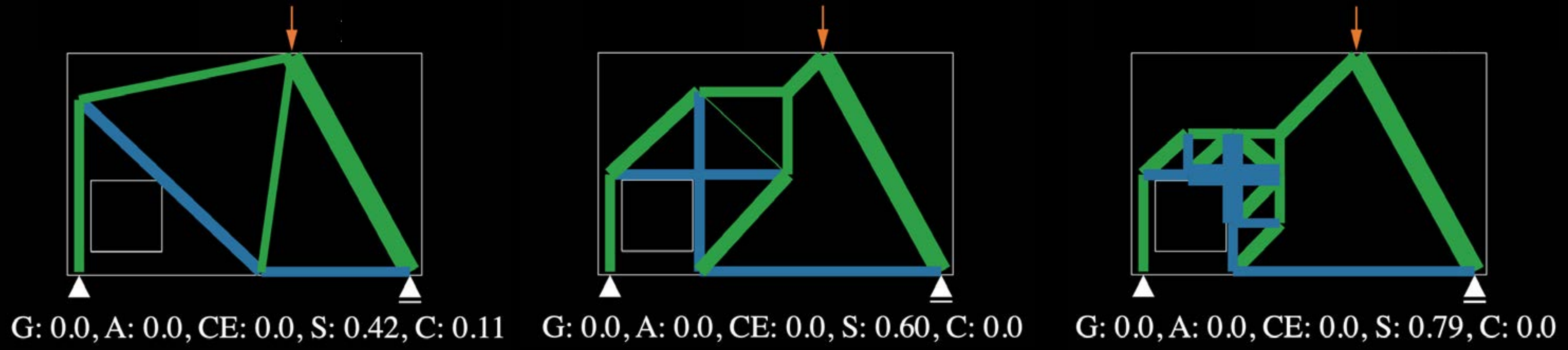


[Yu et al. 2024]



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Future development of stress fields / STM

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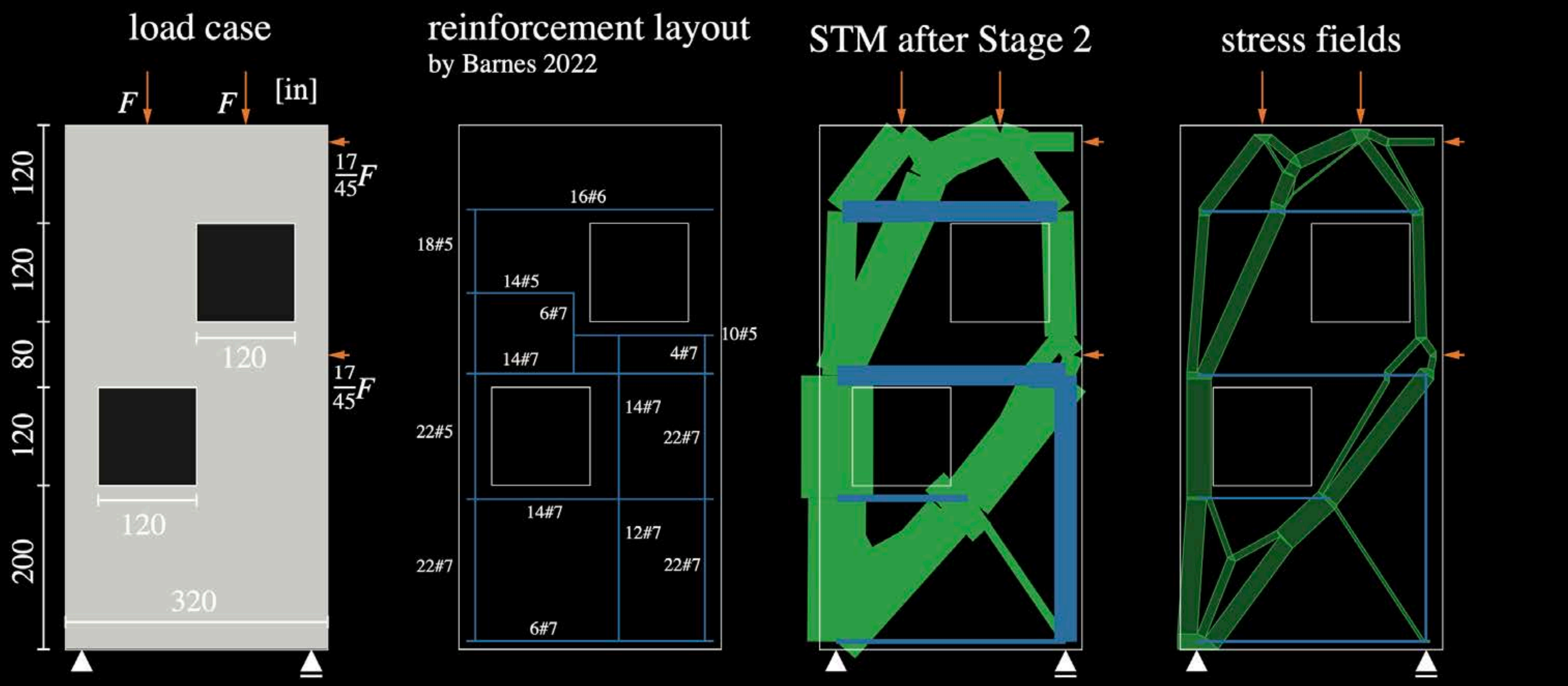
2D Problems

Design:	develop a user-friendly end-to-end computational tool with strut-and-tie models and stress fields (e.g. define reinforcement with an automated strut-and-tie model, validate with CSFM)
Assessment:	develop a user-friendly tool to derive strut-and-tie models based on a given reinforcement layout (e.g. to reconstruct design solution or validate FE-based stress field with interpretable STM)

Generative methods for assessing existing structures

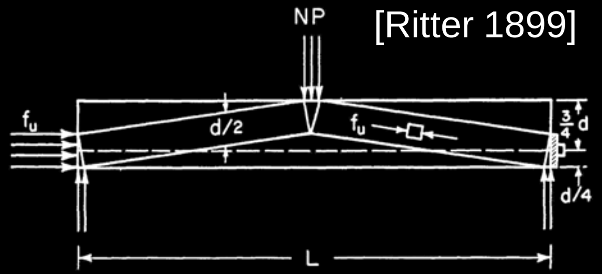
Reverse engineering of STMs



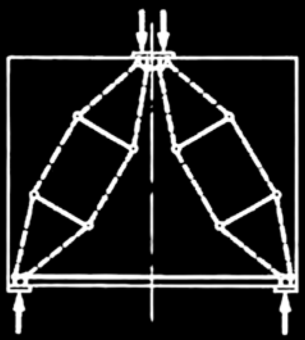
- based on discrete layout optimisation
- ties = existing reinforcement (position, strength)
- maximisation of ultimate load

[Yu et al., “”Reverse engineering strut-and-tie models for assessing reinforced concrete structures” (in preparation)]

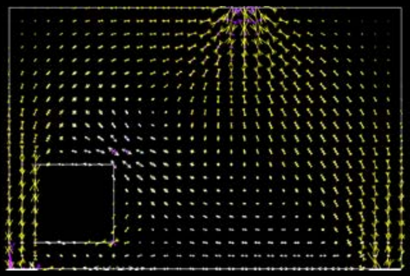
[Ritter 1899]



[Drucker 1961]

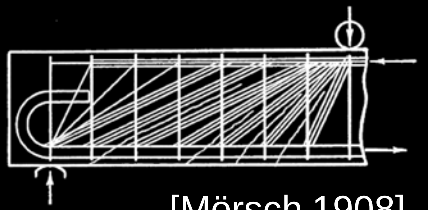


[Schlaich et al. 1987]

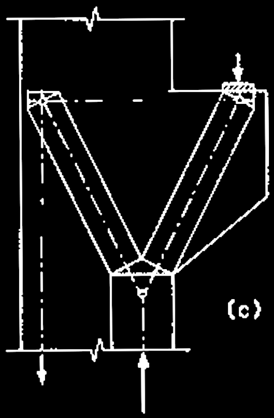


[Kaufmann et al., 2020]

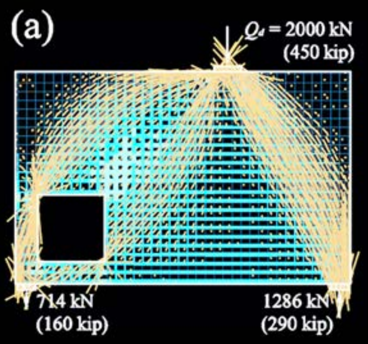
Past **Present** **Future** ?



[Mörsch 1908]



[Marti 1985]



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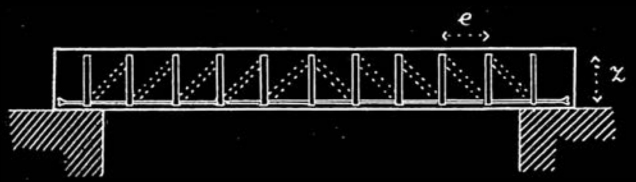

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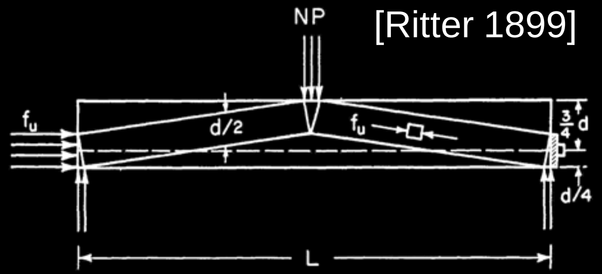
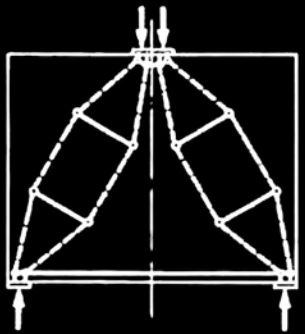


[Drucker 1961]



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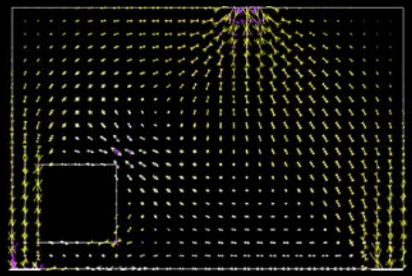

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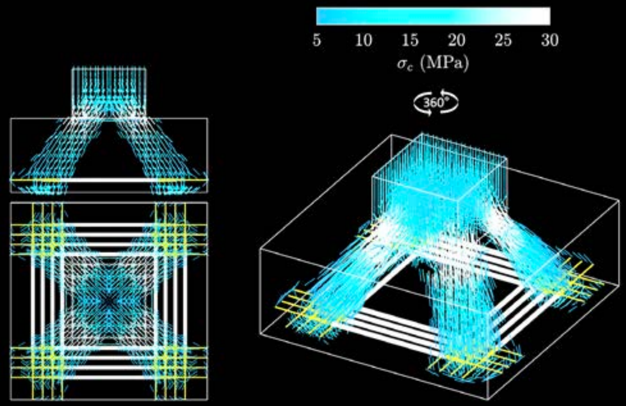
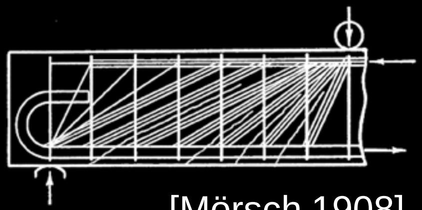


[Meléndez 2017]

Past **Present** **Future**

!



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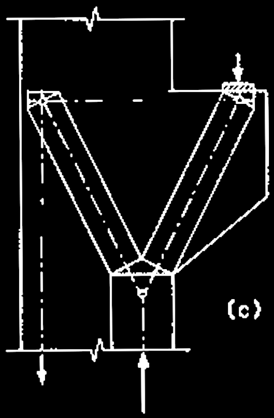

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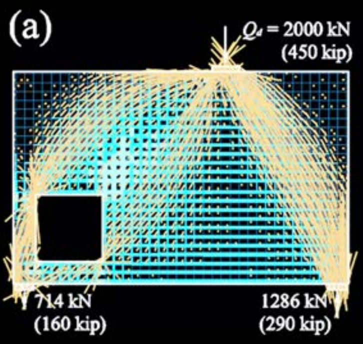

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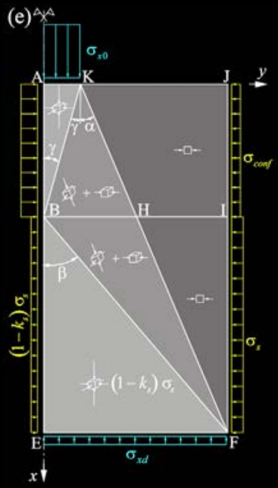
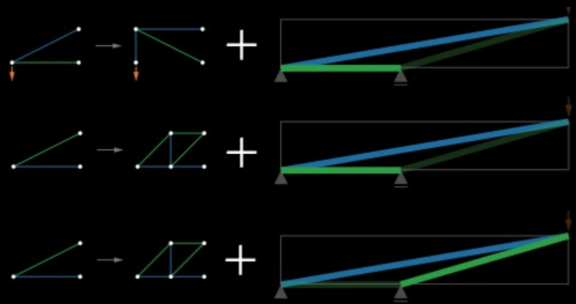
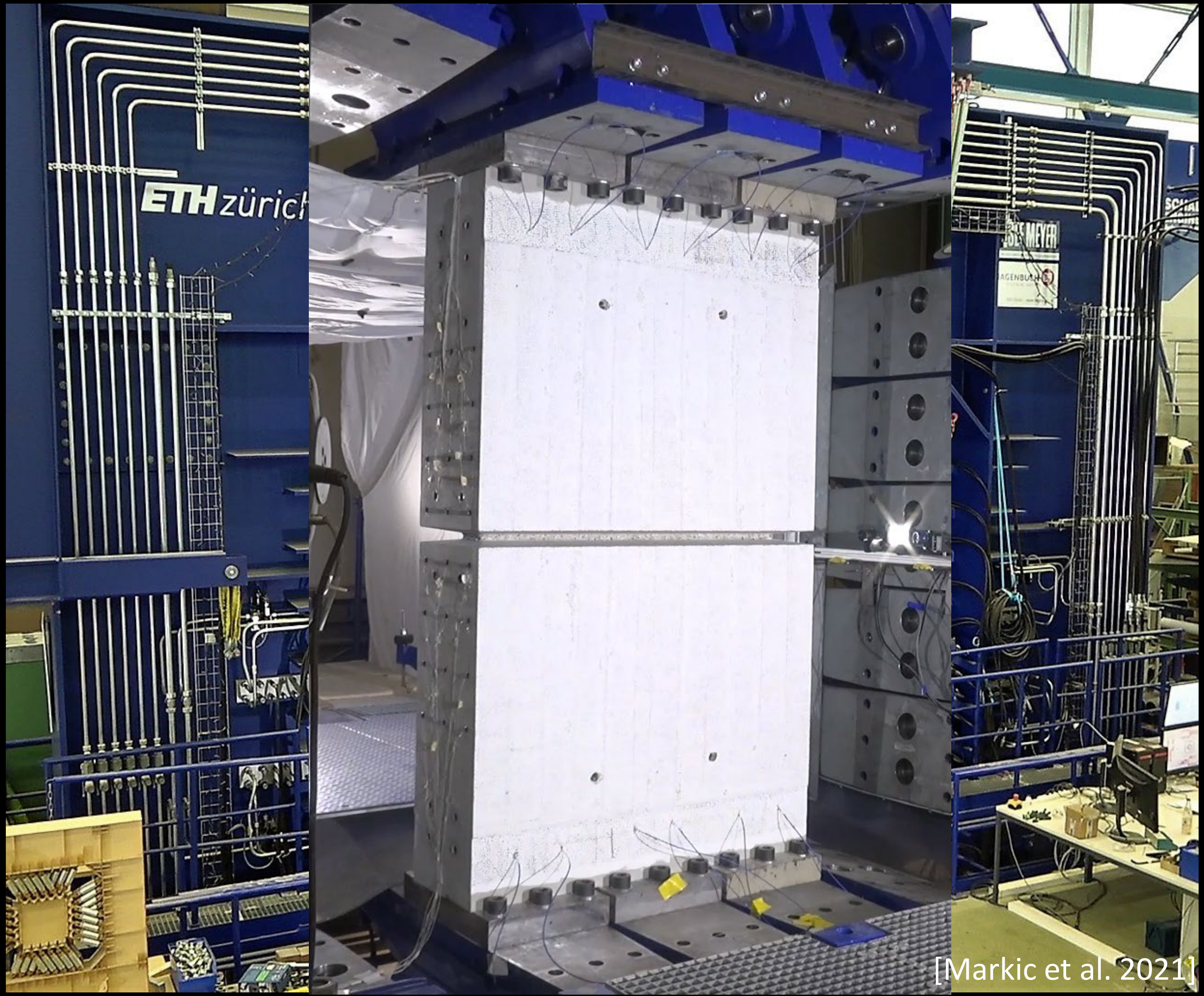


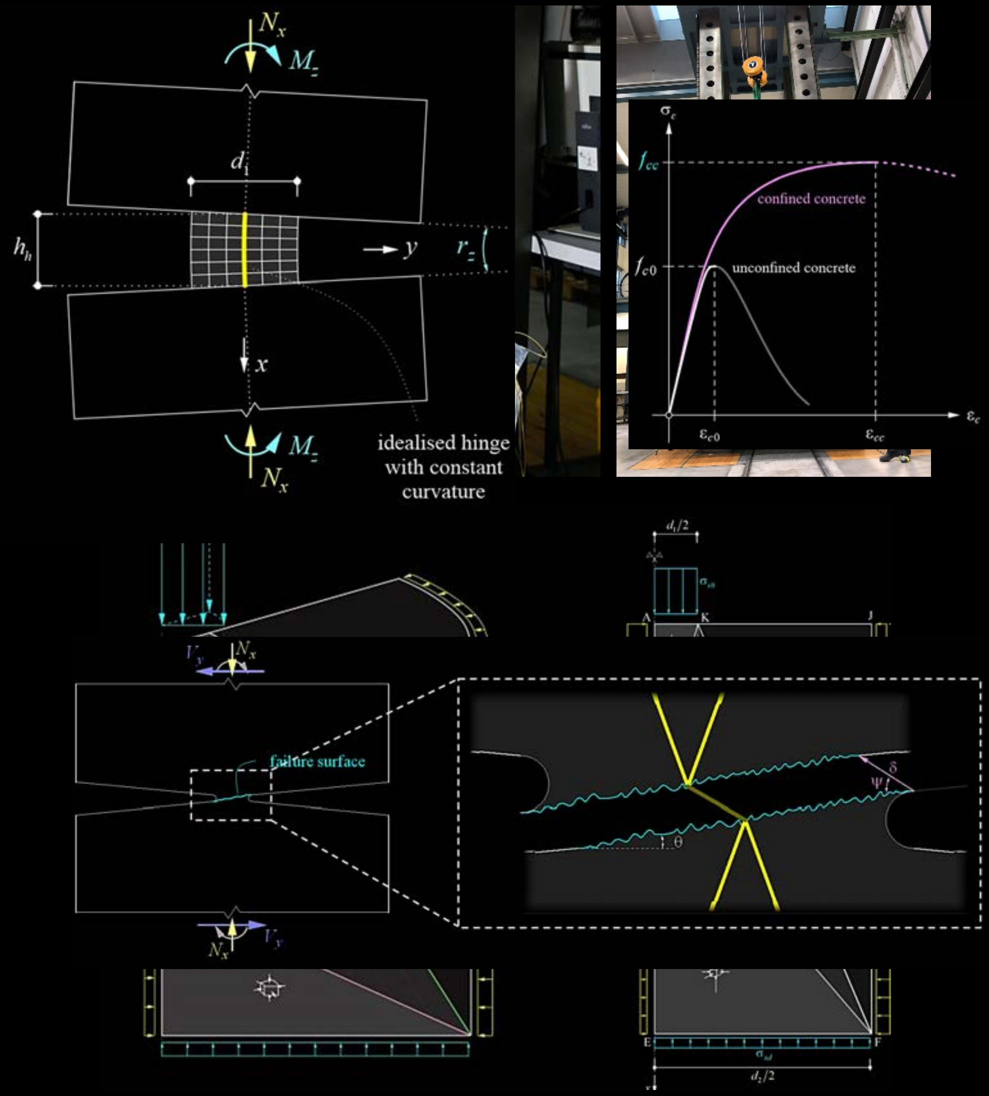
[Markic et al. 2022]



[Yu et al. 2024]

Future developments of stress fields





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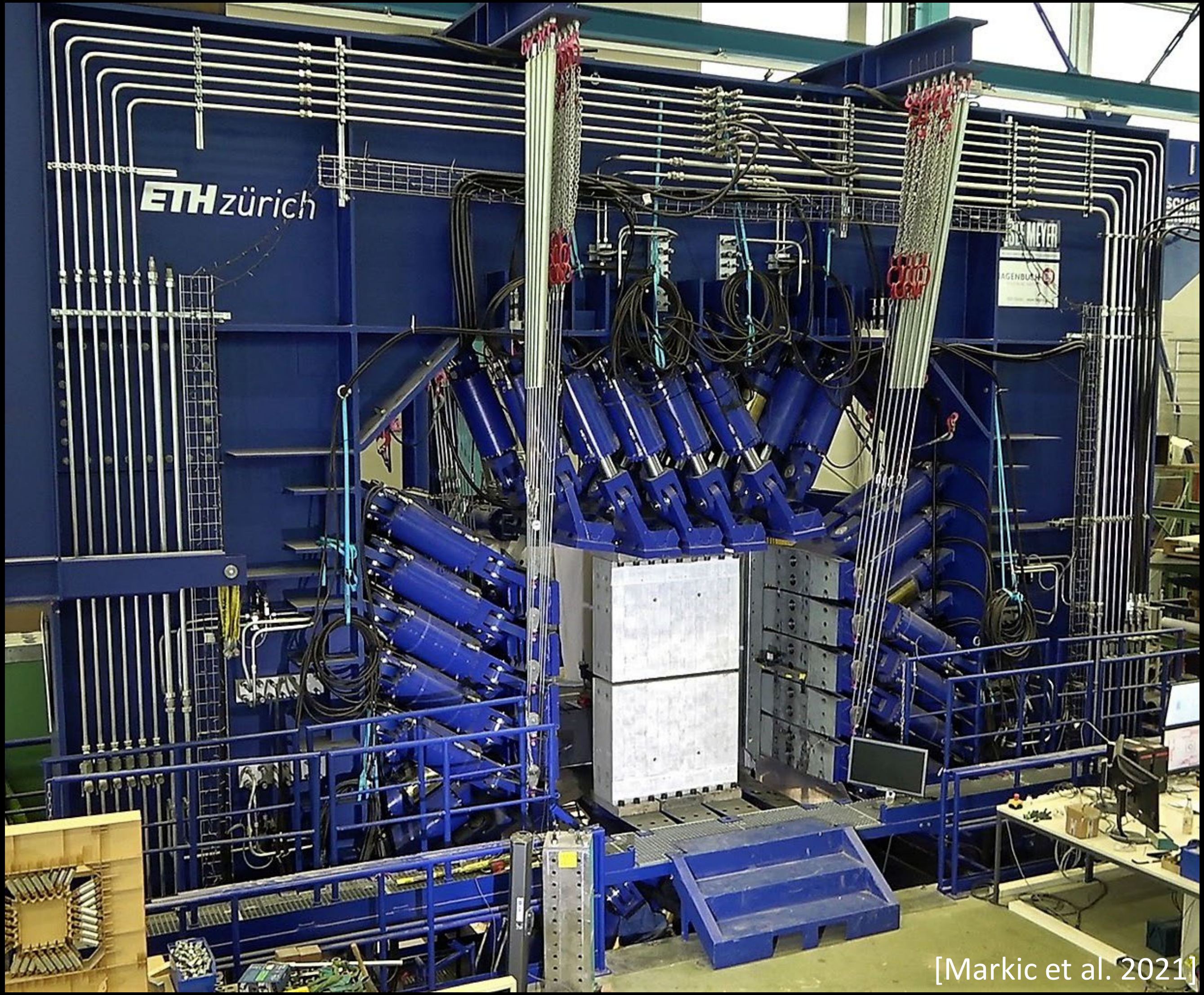

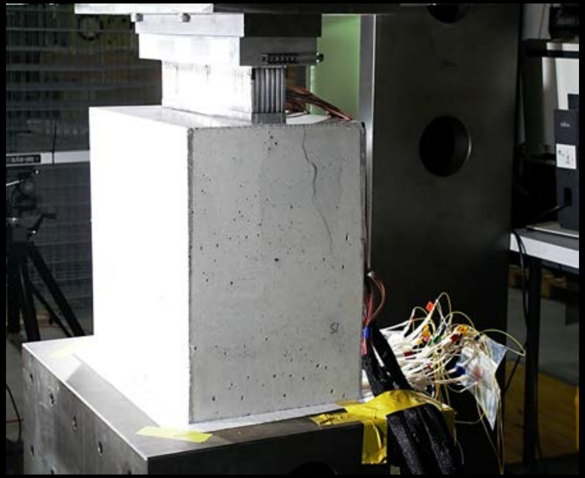

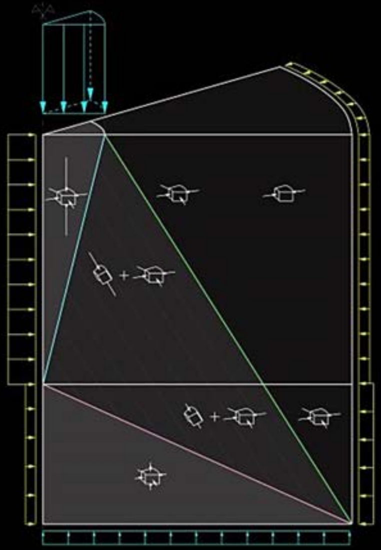

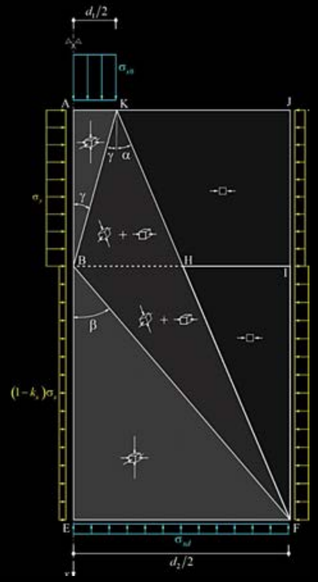

Thank you for the attention

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